

Back to the historical roots of a few combinatorial games

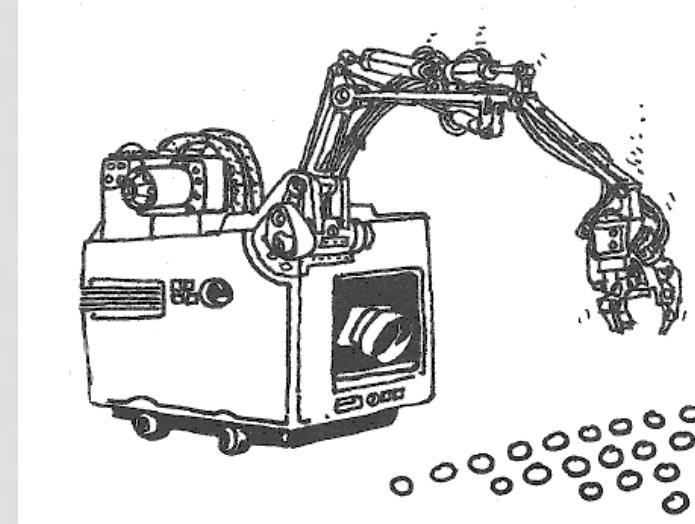
Maastricht University, Digital Ludeme Project
Wednesday, November 6th 2019



Lisa Rougetet
Centre François Viète
Université de Bretagne Occidentale, France

Before we start...

- A definition of a combinatorial game :
 - two players
 - playing alternatively
 - no chance
 - perfect information
 - finite
 - the winner is determined by the last move



Nim game and its prehistoric variants

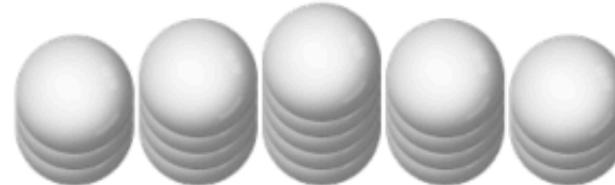
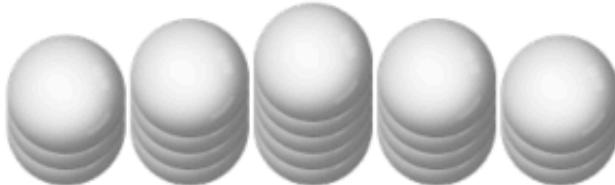


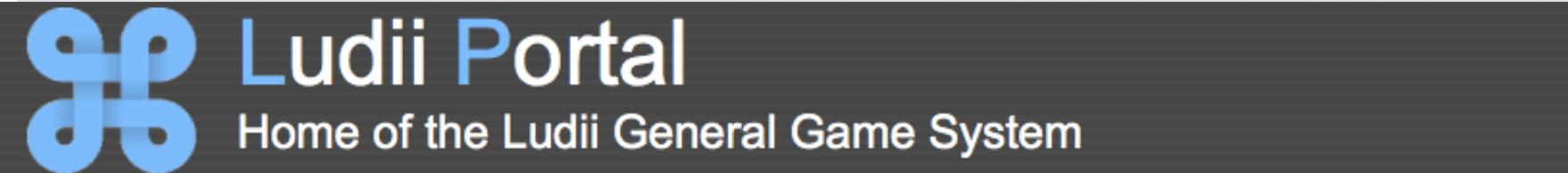
Ludii Portal

Home of the Ludii General Game System

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Nim



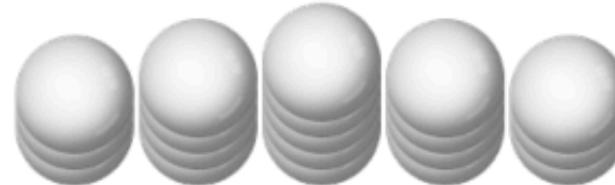
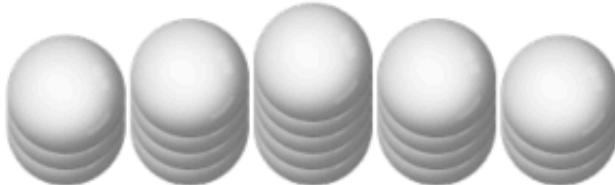


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Nim

Description

Nim is a game of uncertain origins, its earliest mentions in Europe are from the 16th century. It is similar to other games worldwide, but the simple nature of the game indicates that multiple independent inventions may be possible, not necessarily the dispersion of a single game worldwide. Essentially, the game consists of a number of objects in "heaps," and players alternate turns taking any number of objects from one of the heaps. This continues until no objects are left, and the last player to remove an object loses.



Nim game

- Dates back to 1901 (under that name).

Nim game

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NIM, A GAME WITH A COMPLETE MATHEMATICAL THEORY.

By CHARLES L. BOUTON.

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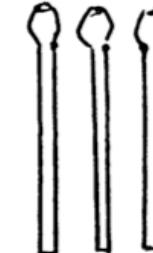
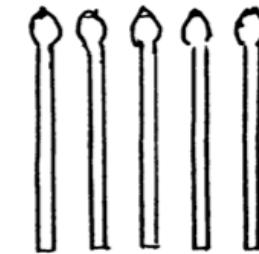
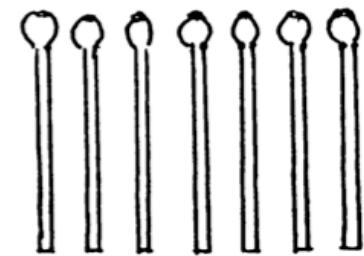
By CHARLES L. BOUTON.

- Published in the *Annals of Mathematics*.
- The word “Nim” comes from the imperative form of the German verb “nehmen”: “nimm”.

NIM, A GAME WITH A COMPLETE MATHEMATICAL THEORY.

By CHARLES L. BOUTON.

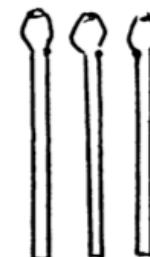
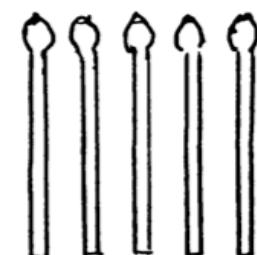
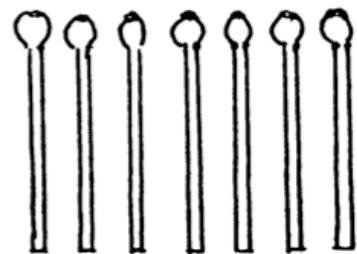
- Played with several piles of matches, each pile contains an arbitrary number of matches.



NIM, A GAME WITH A COMPLETE MATHEMATICAL THEORY.

By CHARLES L. BOUTON.

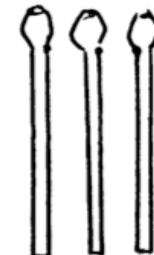
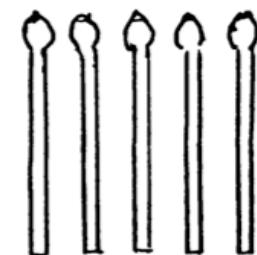
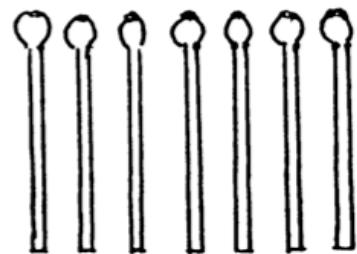
- Played with several piles of matches, each pile contains an arbitrary number of matches.
- At each turn, a player chooses one pile and remove a certain number of matches: one, two, three... or the whole pile.



NIM, A GAME WITH A COMPLETE MATHEMATICAL THEORY.

By CHARLES L. BOUTON.

- Played with several piles of matches, each pile contains an arbitrary number of matches.
- At each turn, a player chooses one pile and remove a certain number of matches: one, two, three... or the whole pile.
- Aim: take the last match(es).

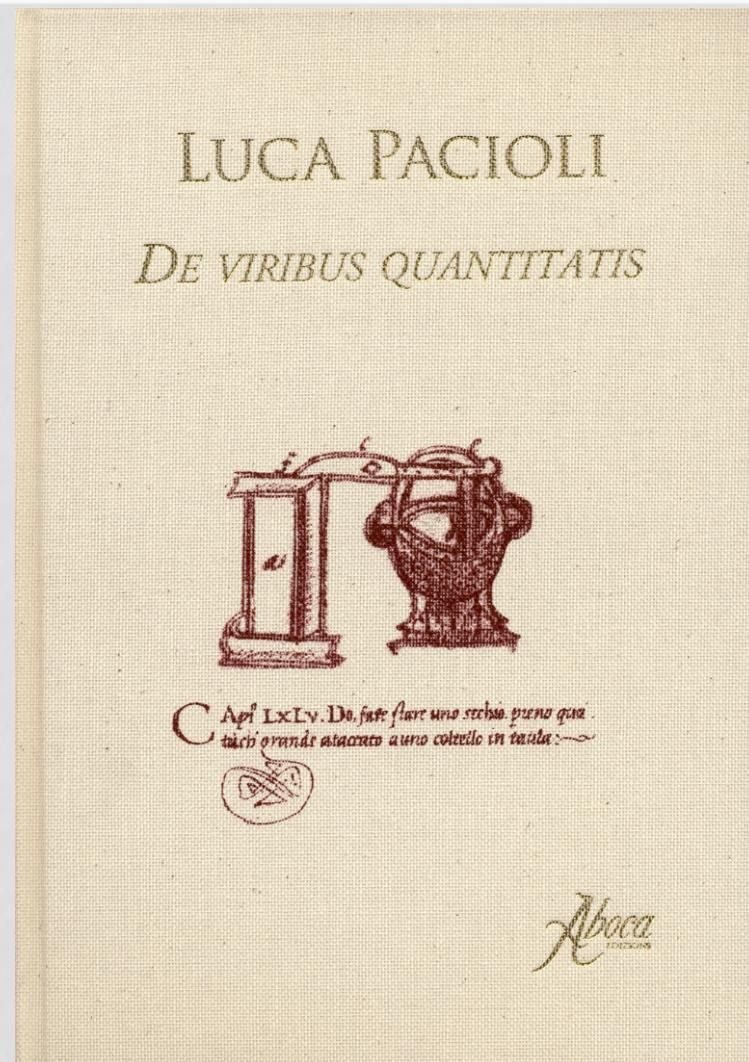


The prehistory of Nim game

The prehistory of Nim game



Luca Pacioli (1445-1517)



Luca Pacioli (1445-1517)

xxxiiii effecto a finire qualunch' numero na
ze al compagno anon prendere piu de un termiato. $\frac{1}{4}$.
Sottra dalse preditte forze non da essere exclusi
alcuni gli giadri guochi honeste et liciti mathema
tici quali communamente se solsano per li corti

Luca Pacioli (1445-1517)

xxxiiii effecto a finire qualunch' numero na
re al compagno anon prendere piu de un termiato. *fi.*
Soriso dase preditte forze non da essere' exclusi
alcuni gli giadri guochi honeste et liciti mathema
tici quali communamente se soliano per si corti

Effecto XXXIIII : How to arrive at any number before your companion and not take more than a determined number.

Luca Pacioli (1445-1517)

*xxxiiii effecto a finire qualunch' numero na
re al compagno anon prendere piu de un termiato. h.
Sottra dalse preditte forze non da essere' exclusi
alcuni gli giadri guochi honeste et liciti mathema
tici quali communamente se solfano per li corti*

Effecto XXXIIII : How to arrive at any number before your companion and not take more than a determined number.

Example: reach the number 30 by summing up, alternately, numbers between 1 and 6.

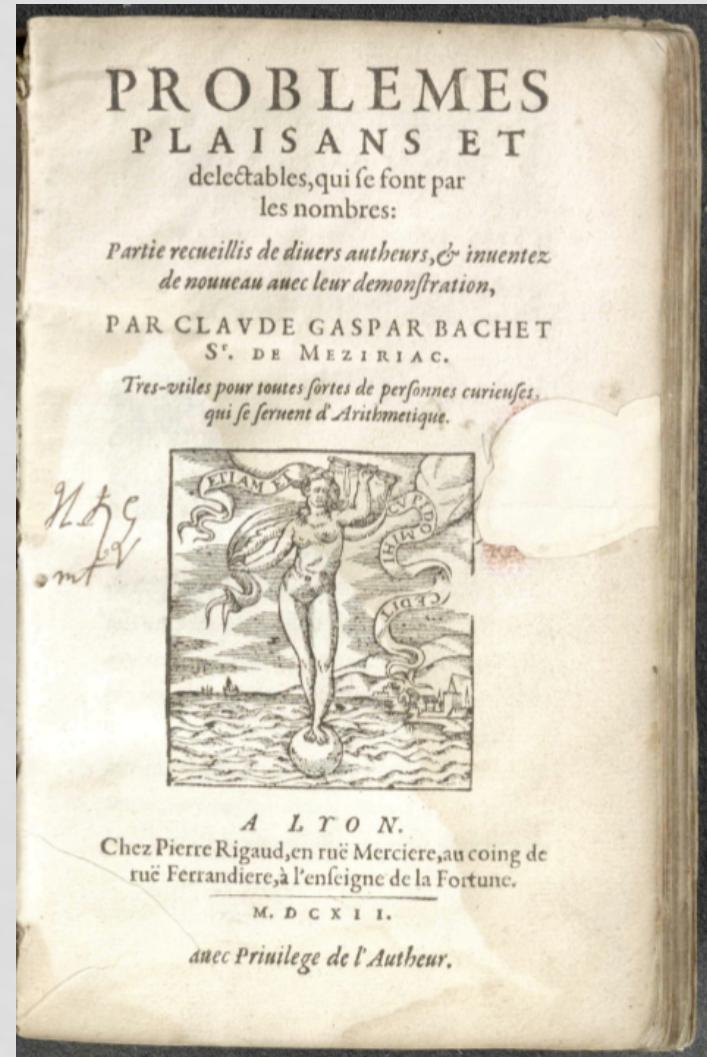
Claude-Gaspard Bachet (1581-1638)



Claude-Gaspard Bachet (1581-1638)



C. G. DE MEZIRIAC. 1635.



A L T O N.
Chez Pierre Rigaud, en rue Mercière, au coing de
rue Ferrandière, à l'enseigne de la Fortune.

M. D C X I I.

avec Privileige de l'Auteur.

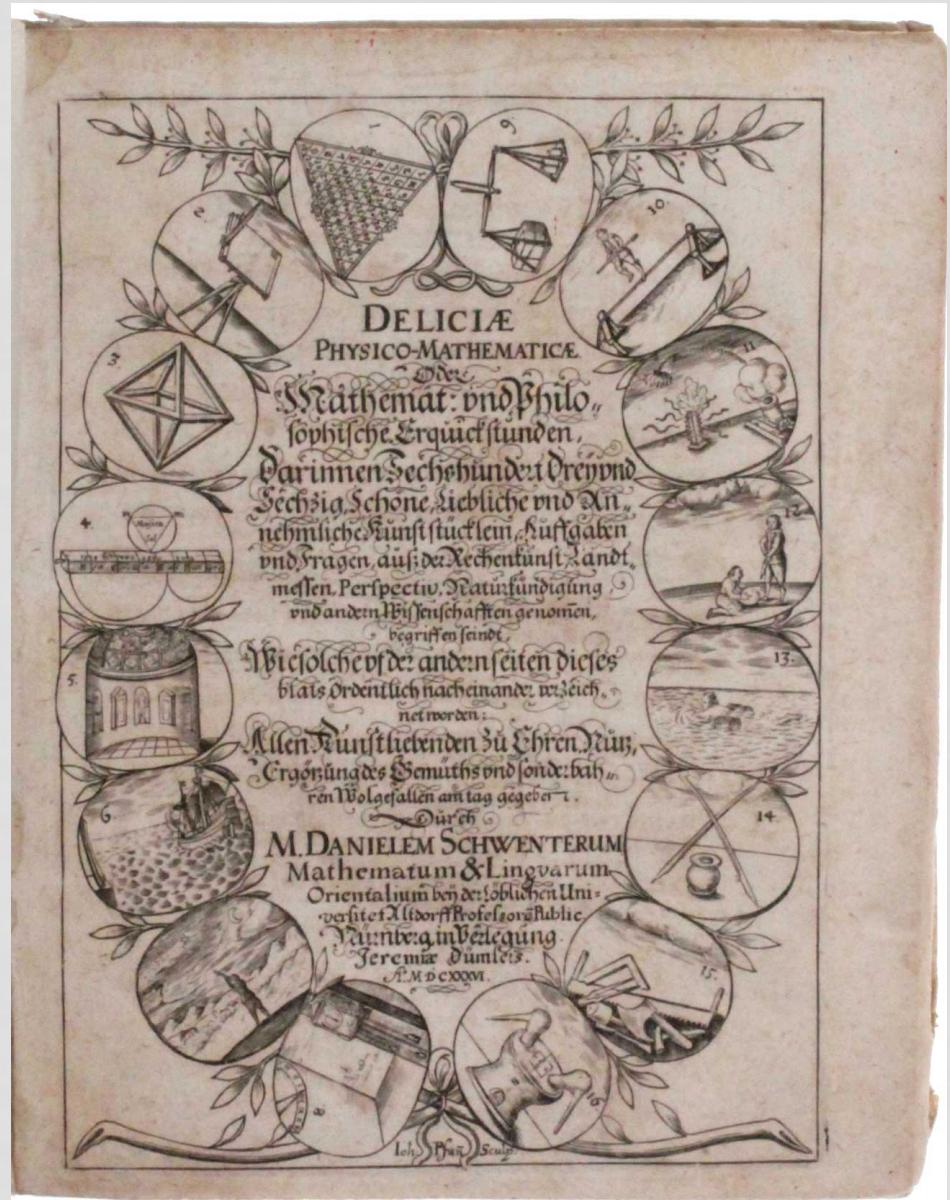
Claude-Gaspard Bachet (1581-1638)

PROBLEME XIX.

Si deux ont proposé entre eux, de dire chascun l'un apres l'autre alternatiuement un nombre à plaisir, qui toutesfois ne surpassé point un certain nombre prefix, pour voir adioustant ensemble les nombres qu'ils diront qui arriuera plusloft à quelque nombre prescrit ; faire si bien qu'on arriue toufiours le premier au nombre destiné.



Daniel Schwenter (1585-1636)





Die XLV. Aufgab.

**Sojßt zween mit einander bis auff 30 zehlen sollen: der gestalt wer am
ersten könne 30 nennen gewonnen habe es soll aber keiner
auff einmahl über 6 zehlen.**

Daniel Schwenter (1585-1636)



Jacques Ozanam (1640-1718)

- French mathematician, teacher, first popularizer of mathematics (so to speak).
- Author of mathematical recreations.

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Bernard de Fontenelle (1657-1757)

Jacques Ozanam

- French mathematician, teacher of mathematics (so to speak)
- Author of mathematical recreations



Bernard de Fontenelle (1657-1757)



ELOGE

DE M. OZANAM.

JACQUES OZANAM n^aquit en 1640 dans la Souveraineté de Dombes d'un Pere riche, & qui avoit plusieurs Terres. La famille étoit d'origine Juive, ce que marque assés le nom , qui a tout-à-fait l'air Hebreu , mais il y avoit long-temps que cette tache , peut-être moins réelle qu'on ne pense , étoit effacée par la profession du Christianisme , & de la Religion Catholique. Cette famille étoit illustrée par plusieurs Charges qu'elle avoit possédées dans des Parlements de Provinces.

M. Ozanam étoit cadet , & par la Loi de son Pays tous les biens devoient appartenir à l'aîné. Son Pere , qui étoit un homme vertueux , voulut réparer ce désavantage par une excellente éducation. Il le destinoit à l'Eglise pour lui faire tomber quelques petits Benefices qui dépendoient de la famille. Les mœurs du jeune Homme étoient bien éloignées de s'opposer à cette destination , elles se portoient naturellement à tout ce qui seroit à désirer dans un Ecclesiastique , & une Mere très pieuse les fortifioit encore & par son exemple & par ses soins , d'autant plus puissants qu'elle étoit tendrement aimée de ce fils. Cependant il ne se tournoit pas volontiers du côté de l'Eglise , il avoit fort bien réussi dans ses Humanités, mais il avoit pris beaucoup de dégoût pour la Philosophie Scolastique, la Theologie ressemblloit trop à cette Philosophie , & enfin il avoit vu par malheur des Livres de Mathematiques, qui lui avoient appris à quoi il étoit destiné.

Il n'eut point de Maître , & on n'avoit garde de lui en donner, mais la Nature seule fait de bons Ecoliers. A 10



LA TRIGONOMETRIE
 RECTILIGNE ET SPHERIQUE;
 AVEC
 LES TABLES
 DES SINUS,
 TANGENTES ET SECANTES;
 Pour un Rayon de 10000000 parties.

ET LES
TABLES DES LOGARITHMES
 DES SINUS
 ET DES TANGENTES,

Pour un Rayon de 10000000000 parties.

Par M. OZANAM, de l'Academie Royale des Sciences;
 tirés de son Cours de Mathématique.

Nouvelle Edition, revue & corrigée exactement.



A PARIS, RUE DAUPHINE,

Chez CH. ANT. JOMBERT, Libraire du Roi pour l'Artillerie &
 le Génie, à l'Image Notre-Dame.

M. DCC. LXV.

Avec Approbation & Privilège du Roi.

2 Degrés.						87 Degrés.						
Minut.	Sinus.	Tangent.	Sécantes.	Log. Sin.		Minut.	Sinus.	Tangent.	Sécantes.	Log. Sin.	Log. Tang.	
3489.95	3492.01	100060.95		8.1421091		3489.95	3492.01	100060.95	2863615.3	2865370.8	9.9997354	11.4569162
3519.02	3521.07	100061.97		8.1464111		3519.02	3521.07	100061.97	2839939.7	2841699.7	9.9997309	11.4533091
3548.09	3550.31	100063.00		8.1539371		3548.09	3550.31	100063.00	2816641.2	2818416.8	9.9997365	11.4497317
3577.16	3579.45	100064.04		8.1570311		3577.16	3579.45	100064.04	2793713.3	2795512.5	9.9997320	11.4461834
3606.23	3608.58	100065.09		8.1601401		3606.23	3608.58	100065.09	2771174.0	2772977.7	9.9997174	11.4436638
3635.30	3637.71	100066.15		8.1632491		3635.30	3637.71	100066.15	2748855.3	2750803.5	9.9997158	11.4391724
3664.37	3666.83	100067.21		8.1663481		3664.37	3666.83	100067.21	2727148.6	2728981.4	9.9997083	11.4357388
3693.44	3695.96	100068.28		8.1694471		3693.44	3695.96	100068.28	2705655.7	2707503.0	9.9997036	11.4322725
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3780.65	3783.35	100071.55		8.1777441		3780.65	3783.35	100071.55	2649928.5	2643160.0	9.9996894	11.4221234
3809.71	3812.48	100072.66		8.1808431		3809.71	3812.48	100072.66	2612963.8	2614869.4	9.9996847	11.4187923
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3867.85	3870.74	100074.89		8.1872411		3867.85	3870.74	100074.89	2559925.7	2558482.3	9.9996745	11.4122095
3896.91	3899.88	100076.02		8.1903401		3896.91	3899.88	100076.02	2529924.4	2526418.3	9.9996696	11.4089491
3925.98	3929.01	100077.16		8.1934391		3925.98	3929.01	100077.16	2499926.0	2545170.0	9.9996650	11.4057168
3955.05	3958.14	100078.31		8.1965381		3955.05	3958.14	100078.31	2469927.5	2514636.1	9.9996601	11.4025083
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4216.63	4220.38	100089.02		8.2244291		4216.63	4220.38	100089.02	2199934.0	2369453.7	9.9996136	11.3746682
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4332.88	4336.95	100094.00		8.2368251		4332.88	4336.95	100094.00	2079937.4	2290376.1		
4361.94	4366.09	100095.27		8.2399241		4361.94	4366.09	100095.27	2049938.2	2292558.6		
30												

Bij

DICTIONNAIRE MATHÉMATIQUE, OU IDÉE GÉNÉRALE DES MATHÉMATIQUES.

DANS LEQUEL SONT CONTENUS LES TERMES
de cette science, outre plusieurs termes des Arts & des autres sciences,
avec des raisonnemens qui conduisent peu à peu l'esprit à une connoissance
universelle des Mathematiques.

Par M. OZANAM, Professeur des Mathematiques
du Roy Tres-Chrétien à Paris.



Sur l'Imprimé à Paris.

A AMSTERDAM,
Aux dépens des HUGUETAN.

M. D. C. LXXXI.

<i>Lignes droites</i>		P. no.
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<i>Parallélogrammes</i>		
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<i>Hexagone</i>	<i>Pentagone</i>	
<i>Solides</i>		
<i>Prisme</i>	<i>Parallélépipede</i>	
<i>Cube</i>	<i>Pyramide</i>	

RECREATIONS MATHÉMATIQUES ET PHYSIQUES,

QUI CONTIENNENT

Plusieurs Problèmes d'Arithmetique, de Géométrie, d'Optique, de Gnomonique, de Cosmographie, de Mécanique, de Pyrotechnie, & de Physique. Avec un Traité nouveau des Horloges Élémentaires.

Par Mr OZANAM, Professeur des Mathématiques.

TOME PREMIER.



V

A PARIS, 1798.

Chez JEAN JOMBERT, près des Augustins,
à l'Image Notre-Dame.

M. DC. XCIV.

AVEC PRIVILEGE DU Roi.

2

V. 1798.

RECREATIONS MATHÉMATIQUES ET PHYSIQUES;

QUI CONTIENNENT

Plusieurs Problèmes d'Arithmetique, utiles & agréables, de Géométrie, d'Optique, de Gnomonique, de Cosmographie, de Mécanique, de Pyrotechnie, & de Physique. Avec un Traité nouveau des Horloges Élémentaires.

Par Mr OZANAM, Professeur des Mathématiques.

TOME SECOND.



V. 1798.
3.

A PARIS,
Chez JEAN JOMBERT, près des Augustins,
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V

18299
V. 1848.
3.

RECREATIONS
MATHÉMATIQUES

RECREATIONS
MATHÉMATIQUES

110 RECREAT. MATHÉMAT. ET PHYS.
PROBLÈME XXI.

Deux personnes étant convenus de prendre à plaisir des nombres moindres qu'un nombre proposé, en continuant alternativement jusqu'à ce que tous leurs nombres fassent ensemble un nombre déterminé plus grand que le proposé, faire qu'on arrive à ce nombre déterminé plus grand.

Chez JEAN JOMBERT, près des Augustins,
à l'Image Notre-Dame.

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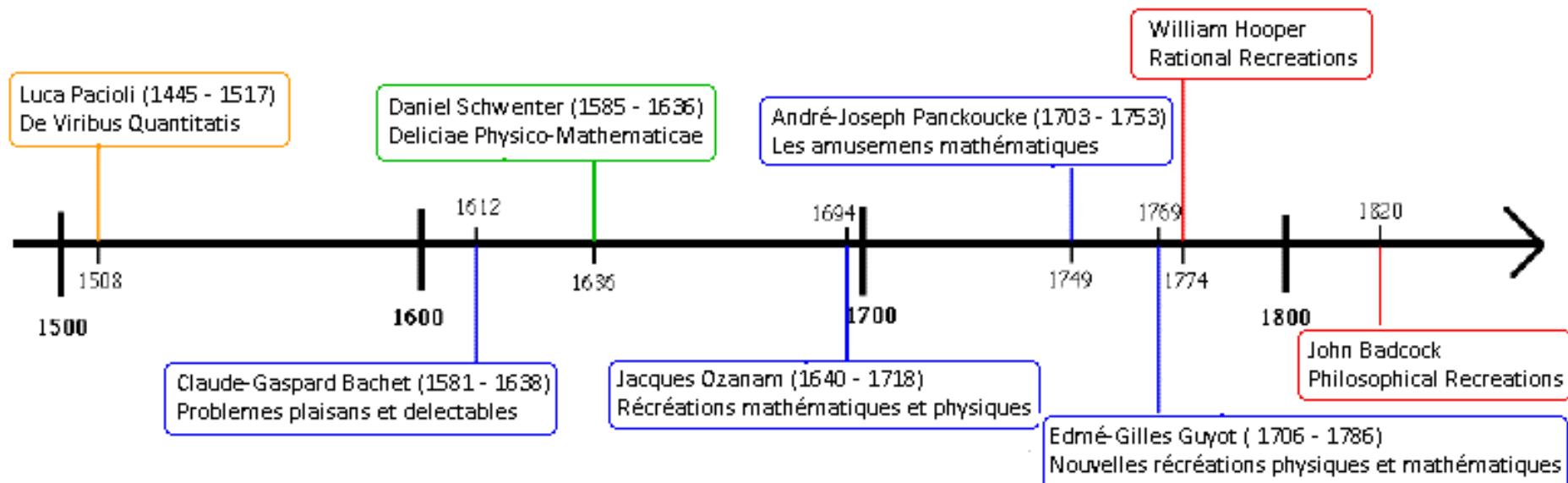
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V. 1998.

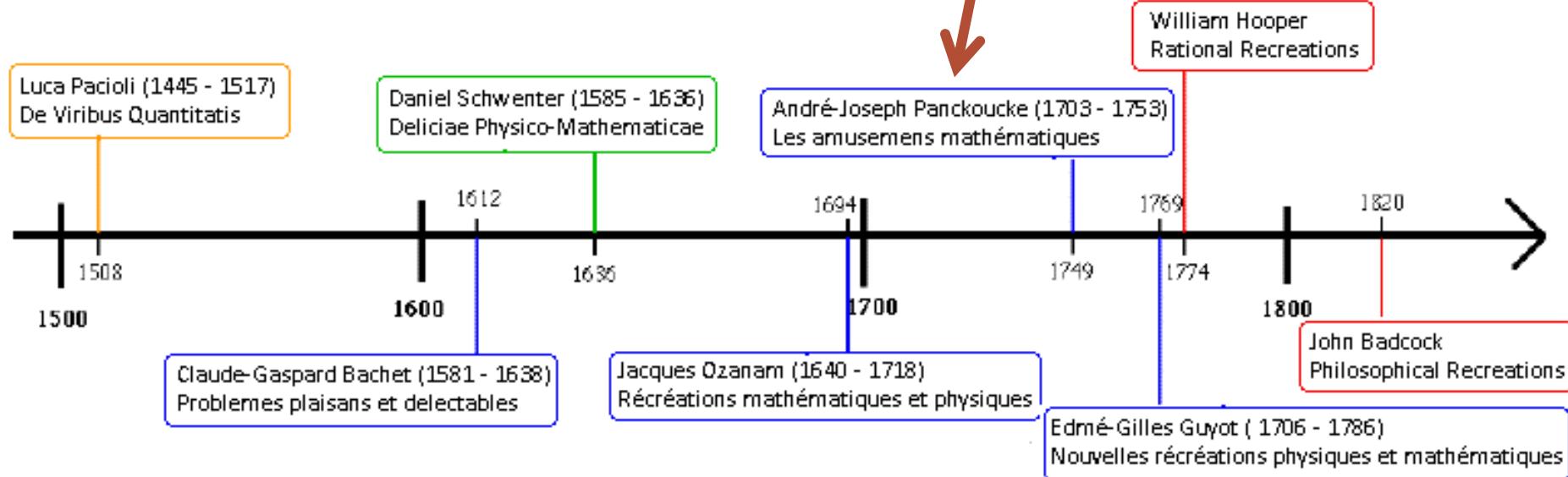
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AVEC PRIVILEGE DU ROI.

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V. 1948.





PROBLEME 10.

Le Piquet des Cavaliers.

Deux amis voyagent à cheval, l'un propose à l'autre un cent de Piquet sans carte.

Ils conviennent, 1°. Que celui qui arrivera le premier à 100 sera défrayé du soupé, 2°. qu'ils ne pourront prendre alternativement plus de 10 à la fois.

LES AMUSEMENS MATHÉMATIQUES

P R E C E D É S

Des Elémens d'Arithmétique, d'Algébre & de Géométrie nécessaires pour l'intelligence des Problèmes.

Sapientem decet interdum Remittere aciem rebus agendis intentam. Aug. de Mus.



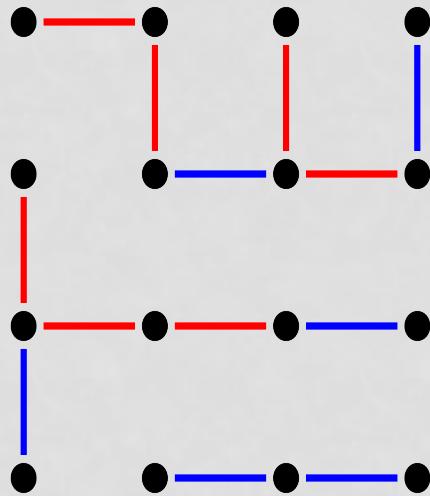
A L I L L E,
Chez ANDRÉ-JOSEPH PANCKOUCKE,
ET SE VEND A PARIS,
Chez TILLIARD, Libraire, Quai des
Augustins, près le Pont Saint-Michel,
à S. Paul.

M. DCC. XLIX.
Avec Approbation & Privilége du Roi.



Conclusion

- Nim game ancestors are part of “mathematical recreations”, a new literary and editorial genre that emerged in the 17th century.
- Particular solutions are provided (not general) with examples.
- The game spread thanks to many reprints of the books (especially Ozanam’s one).



Dots and Boxes and other *jeux de combinaisons* of the 19th c.

DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot

DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot

The image shows the front cover and back cover of a 'Dot to Dot' activity book. The front cover features a grid of blue dots and the title 'Dot to Dot' in large, stylized letters. The back cover provides instructions for the game:

Example

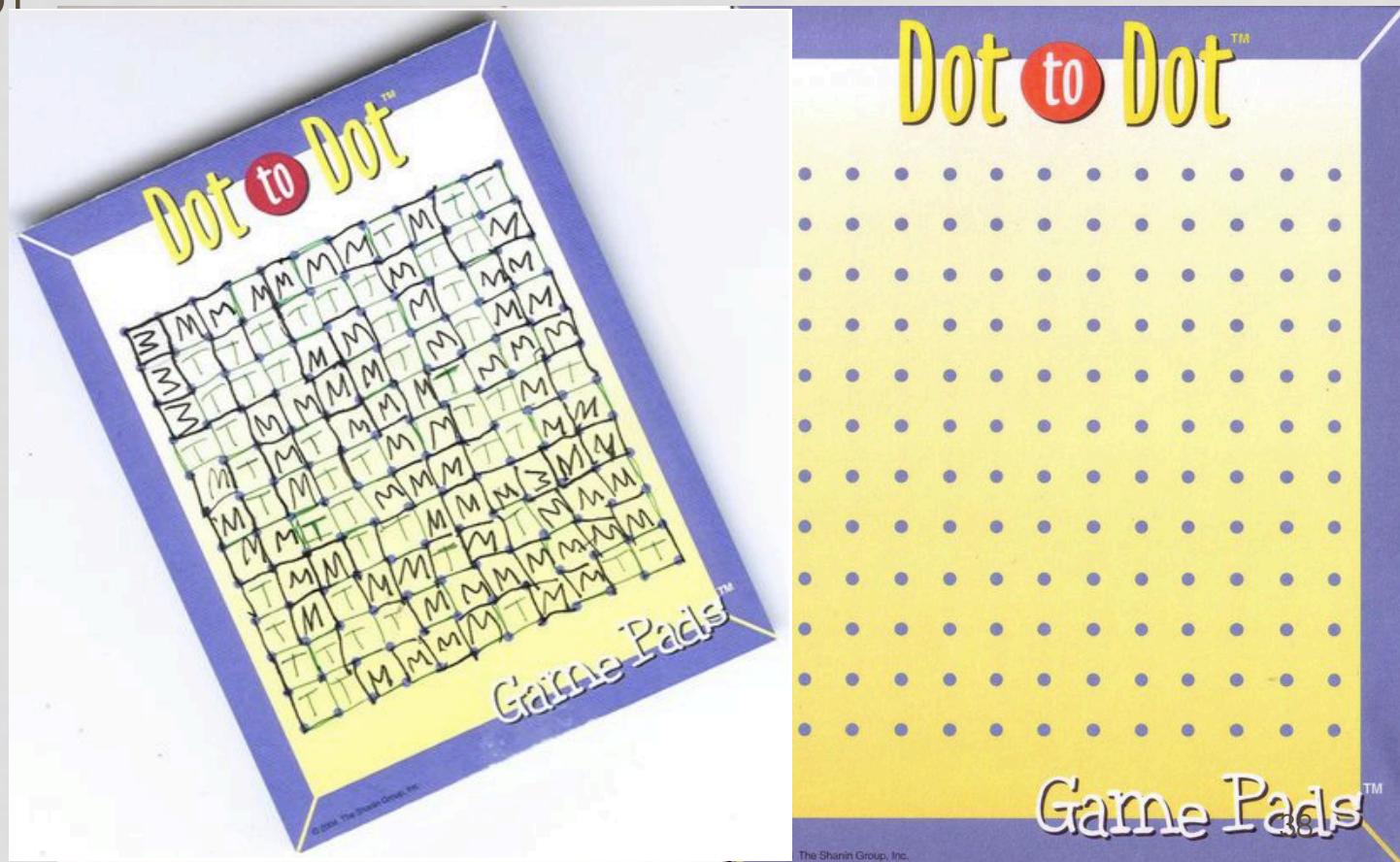
HOW TO PLAY
Dot to Dot™
Enjoy hours of fun with this classic game.
The game is played by two or more people.
Players take turns drawing a line connecting two dots that are next to each other.
The player that makes the final line to complete a box wins that box, and places his or her initial inside.
After winning a box, the player is given another turn.
If a player does not win a box, then it becomes the next player's turn.
The player with the most boxes containing his or her initial wins the game.
50 sheets per pad
© 2004 The Shainin Group, Inc. • Chicago, IL 60659
Made in China

The back cover also features a large grid of blue dots for playing the game.

DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot



DOTS AND BOXES

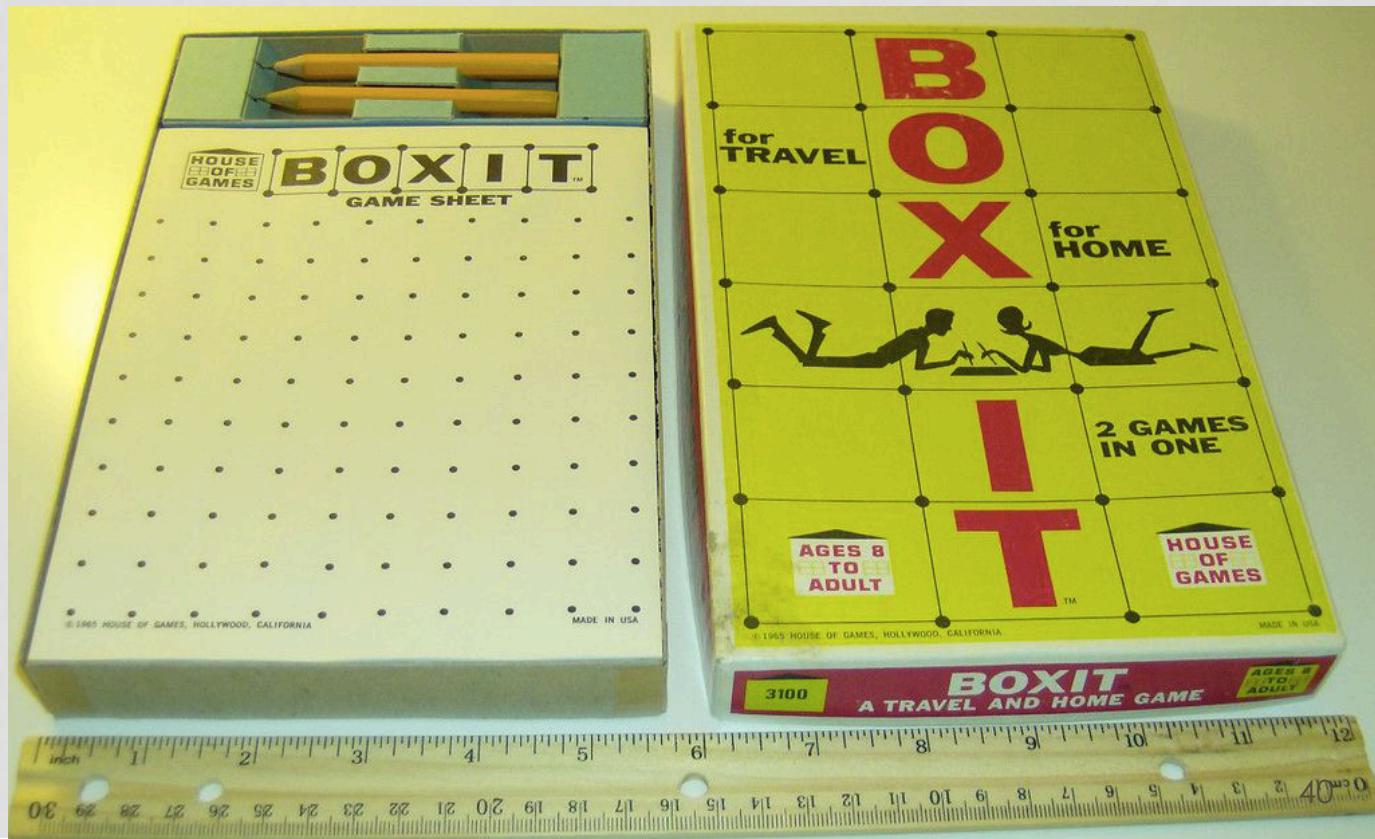
Dots and Boxes is also known as :

- Dot to Dot
- Box It

DOTS AND BOXES

Dots and Boxes is also known as :

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- Box It



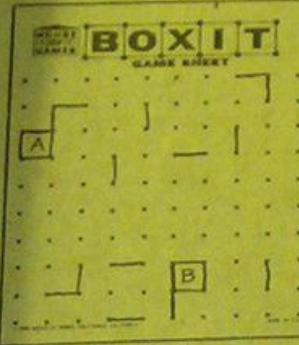
Dots and Boxes

- Dot to Dot
- Box It

NUMBER OF PLAYERS: Two or more may play.

OBJECT OF GAME: To complete the most boxes.

HOW TO PLAY:



(Fig. 1)

Use one game sheet only. Each player, in turn, draws a vertical or horizontal line between two dots next to one another. (Fig. 1) Avoid putting the third side on any possible box, so that an opponent cannot complete a box. When a player completes a box he puts his initial in it to show that he won the box. For each box you complete you get an extra turn.

SCORING:

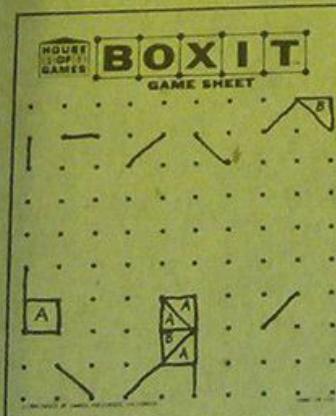
Player with the most boxes wins.

HOW TO PLAY BOXIT-TRIANGLES

NUMBER OF PLAYERS: Two or more may play.

OBJECT OF GAME: To score most points.

HOW TO PLAY:



(Fig. 2)

Use one game sheet. Follow same rules as for BOXIT, except that players can also make diagonal line between two adjacent dots. (Fig. 2) This is a more difficult way to play BOXIT, because now a player must avoid putting either the second side on a possible triangle as well as the third side on a possible box. If a box is completed without a triangle, a triangle may not be added. When the game sheet is filled the game ends.

SCORING:

Each triangle is 5 points. Each box is 5 points. If the same player wins two triangles that make a box he gets 5 points for each triangle and 10 points extra for the box. Each player adds his score and the one with most points wins.

Enjoy BOXIT™ Then play 6 STEPS™ and UPSCOPE™
other Fun Games by the HOUSE OF GAMES

© 1965, House of Games, Hollywood, California

DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot
- Box It
- Squares

DOTS AND BOXES

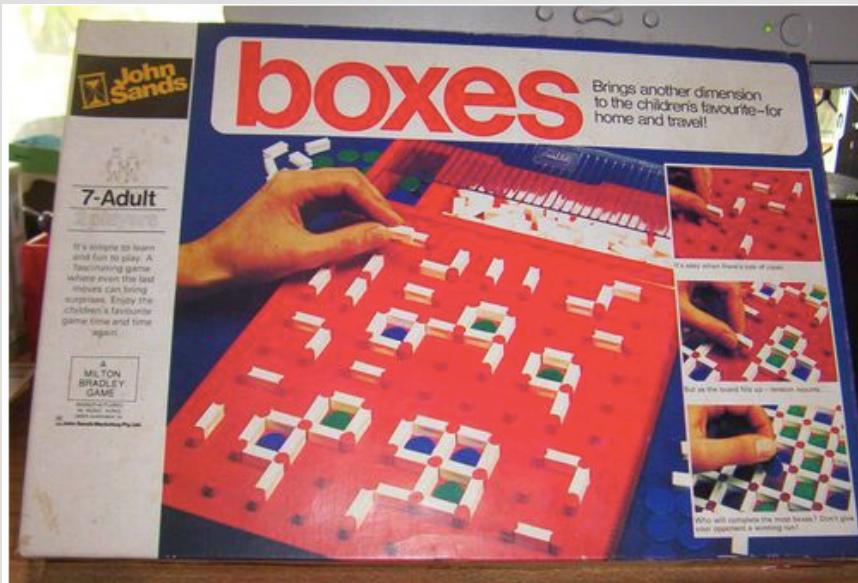


DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot
- Box It
- Squares
- Boxes (Kamertje verhuren (BE), Square (DE), Les petits carrés (FR) ...)

DOTS AND BOXES



DOTS AND BOXES

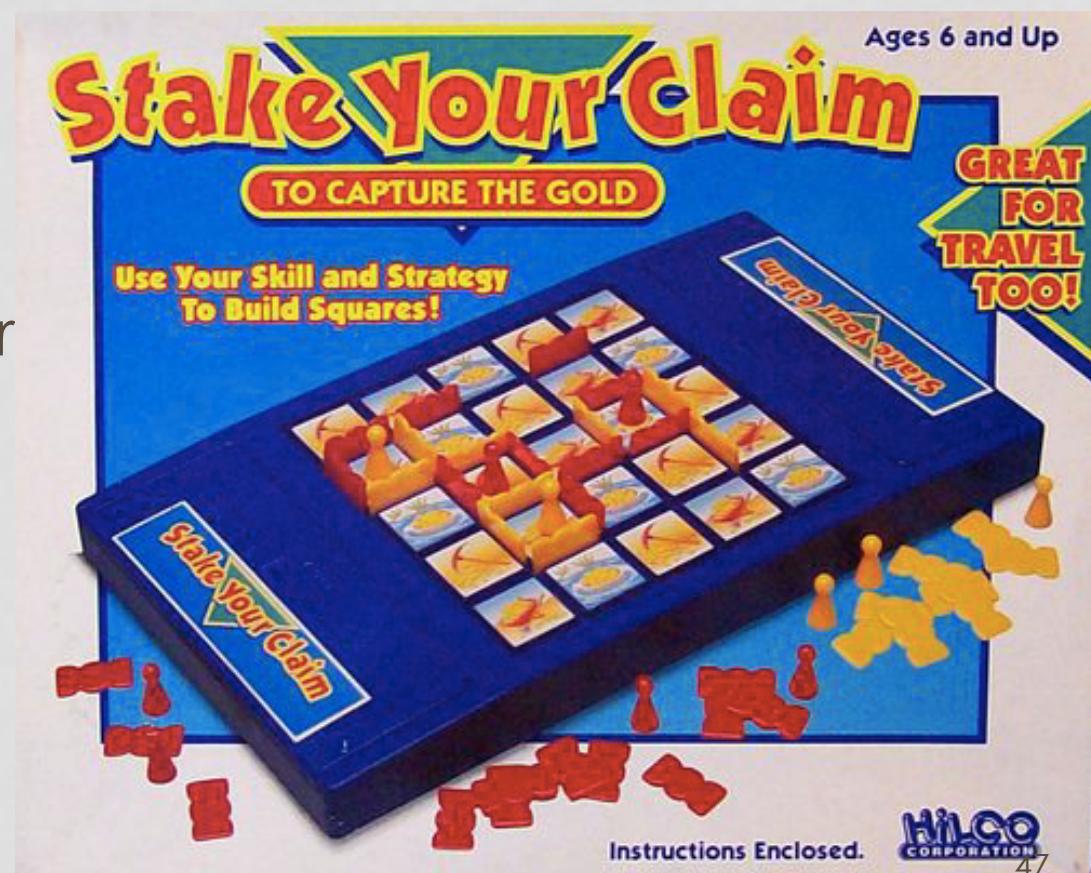
Dots and Boxes is also known as :

- Dot to Dot
- Box It
- Squares
- Boxes (Kamertje verhuren (BE), Square (DE), Les petits carrés (FR) ...)
- Stake Your Claim

DOTS AND BOXES

Dots and Boxes is also known as :

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- Squares
- Boxes (Kamertje ver carrés (FR) ...)
- Stake Your Claim



DOTS AND BOXES

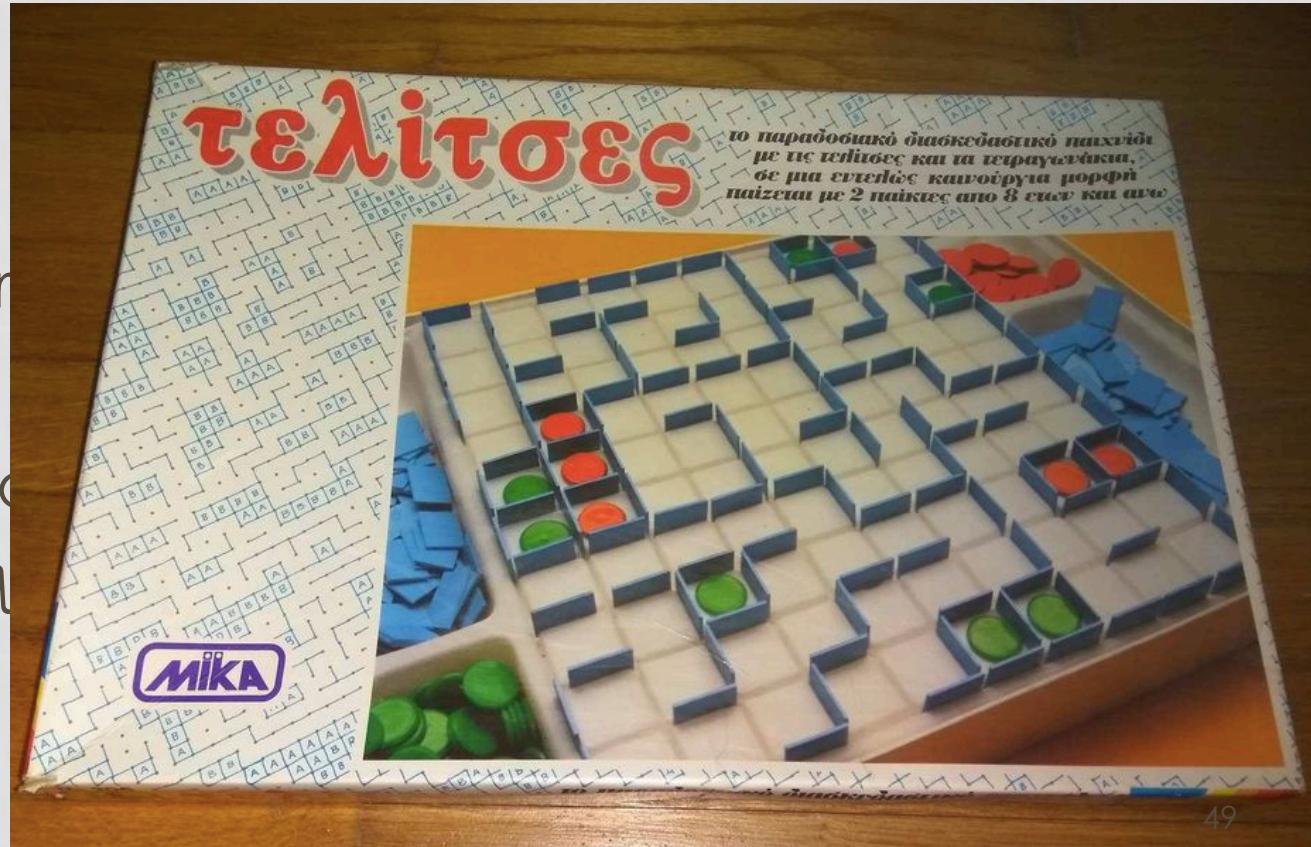
Dots and Boxes is also known as :

- Dot to Dot
- Box It
- Squares
- Boxes (Kamertje verhuren (BE), Square (DE), Les petits carrés (FR) ...)
- Stake Your Claim
- Telitses (τελίτσες)

DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot
- Box It
- Squares
- Boxes (Kamer...
carrés (FR) ...)
- Stake Your Claim
- Telitses (τελίτσες)



DOTS AND BOXES

Dots and Boxes is also known as :

- Dot to Dot
- Box It
- Squares
- Boxes (Kamertje verhuren (BE), Square (DE), Les petits carrés (FR) ...)
- Stake Your Claim
- Telitses (τελίτσες)
- and many others...

The original game/name...

The original game/name...

LA PIPÉPIPEGÉE
JEU DE COMBINAISONS

Dédicé aux Élèves de l'X
PAR UN CANTIQUE



EDOUARD LUCAS (1842 - 1891)

French arithmetician.
He published a lot of
works on arithmetics
and its applications,
such as recreational
mathematics.



Le jeu de l'école polytechnique

« nouveau jeu de combinaison assez original »

« *a new game of combination, quite original* »

« Ce jeu nous a paru assez curieux pour en donner ici la description ; mais jusqu'à présent, nous ne connaissons pas d'observations, ni de remarques assez importantes pour en dire davantage. »

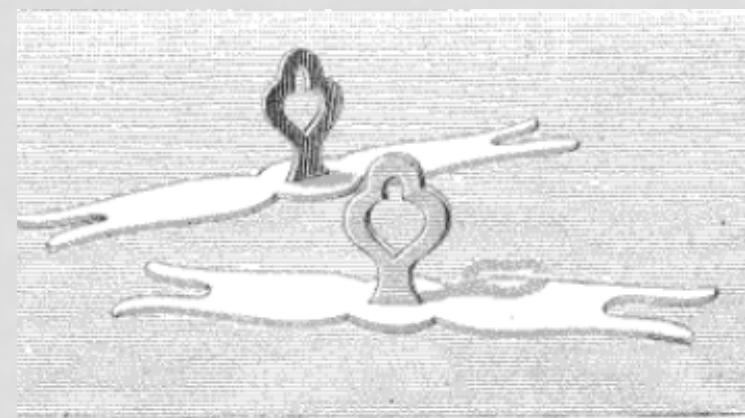
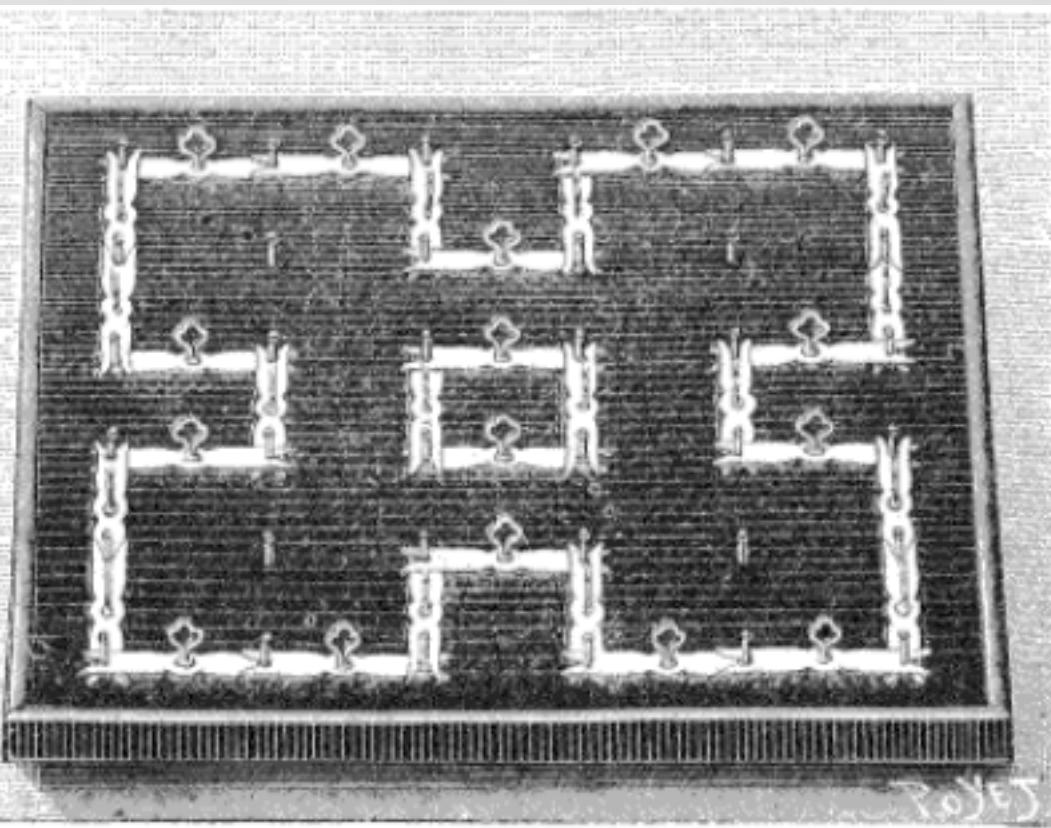
« *This game seemed curious enough to give the description here; but so far, we do not know any comments or remarks important enough to say more.* »

La Pipopipette

« La Pipopipette est un jeu fort original dont les combinaisons ne laissent rien au hasard et qui peut se jouer à deux, trois, quatre personnes ou plus. [...] elle a été imaginée à l'École Polytechnique par plusieurs de mes anciens élèves de Spéciales. C'est donc avec toute justice que je suis heureux de leur dédier ce jeu. [...] En la voyant, il s'écrieront : Ah ! saperlipopette ! Belle Pipopipette ! »

« The Pipopipette is a very original game whose combinations leave nothing to chance and can be played by two, three, four or more people. [...] it was imagined at the École Polytechnique by several of my former students of Spéciales. It is with all justice that I am happy to dedicate this game to them. [...] On seeing it, they will exclaim: Ah! saperlipopette! Beautiful Pipopipette! »

La Pipopipette



Overall view of the board (5 by 5) and detailed bars that connect two dowels.
In: *La Nature*, n°853, 5 Octobre 1889, *Nouveaux jeux scientifiques de M. E. Lucas*

GERMAN PATENT 1899

ROBERT MARQUARD UND GEORG FRIECKERT IN BERLIN.

Gesellschaftsspiel.

Fig. 1.

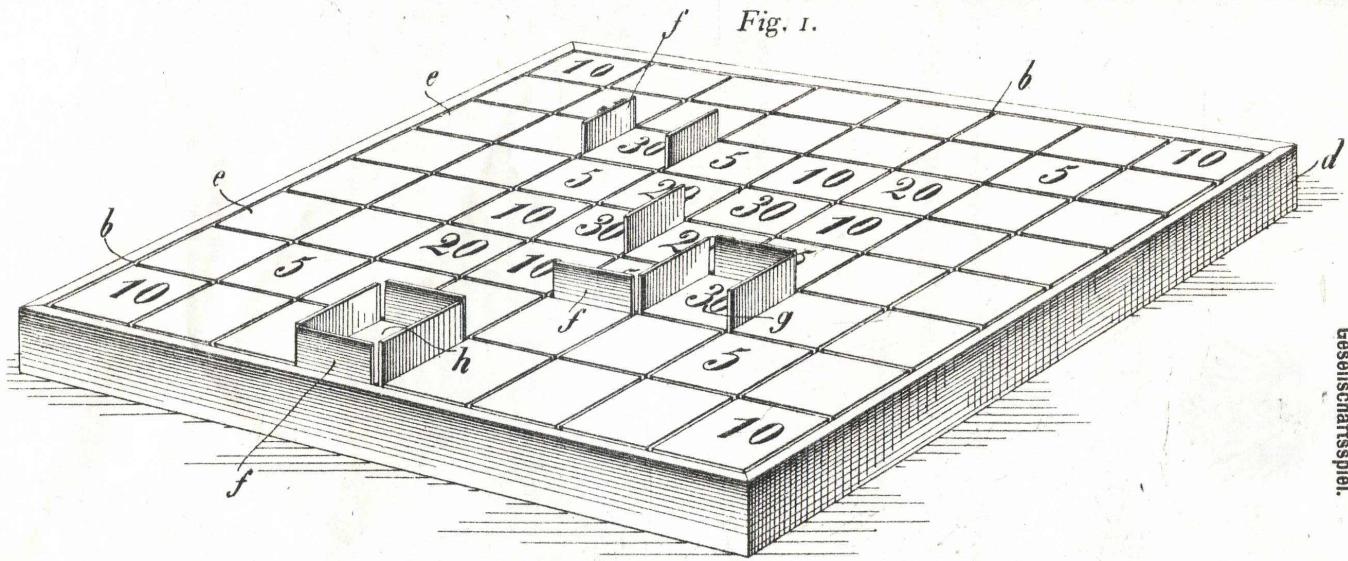
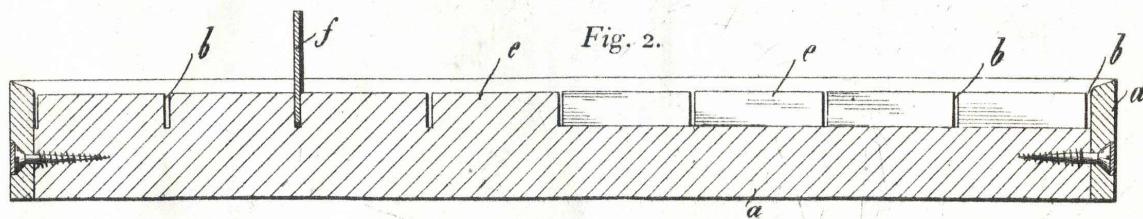


Fig. 2.



M 10883.

Zu der Patentschrift

PHOTOGR. DRUCK DER REICHSRUCKEREI

GERMAN PATENT 1899

ROBERT MARQUARD UND GEORG FR.

Gesellschaftsspiel.

Fig. 1.

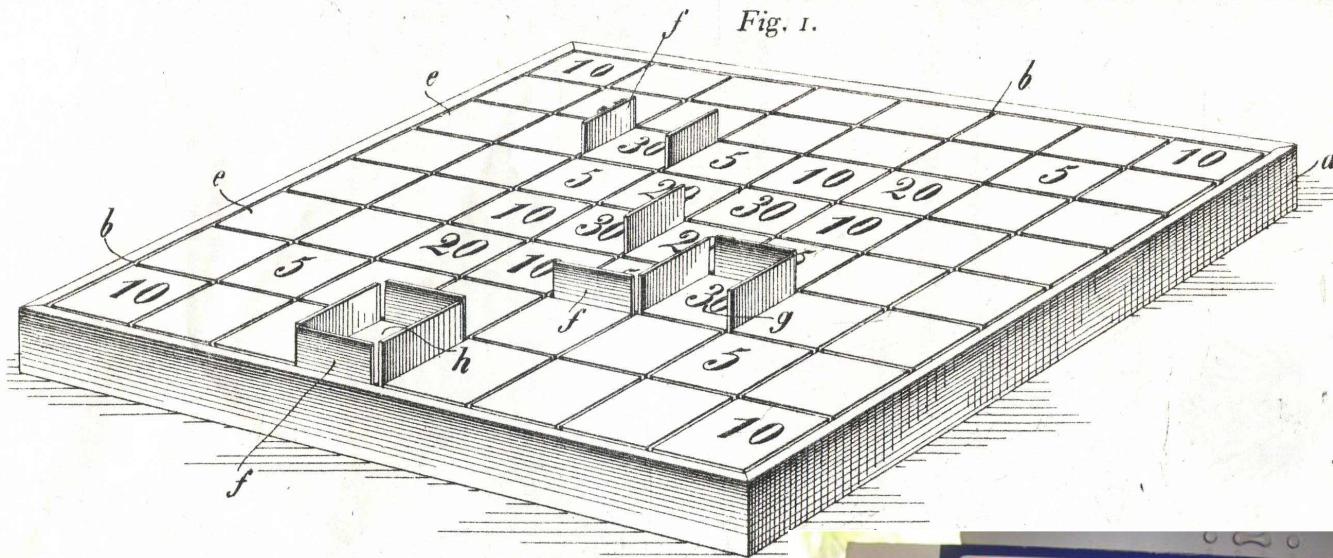
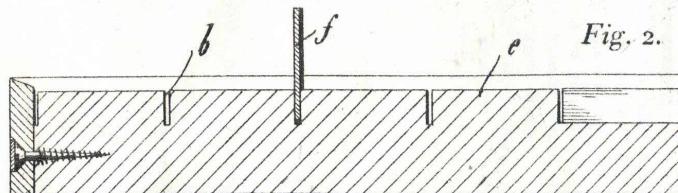


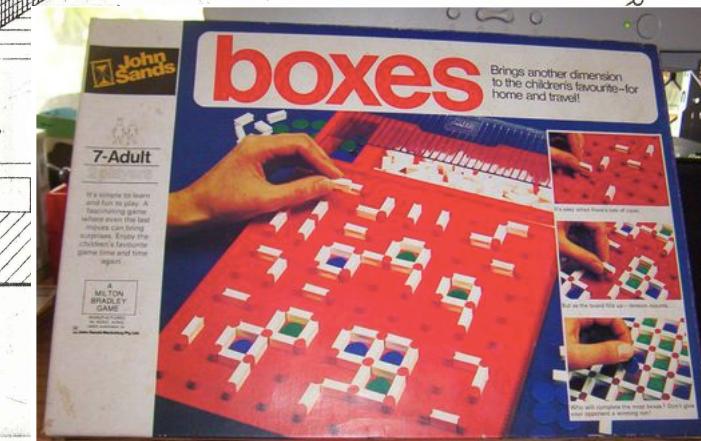
Fig. 2.



M 10883.

Zu der Patentschrift

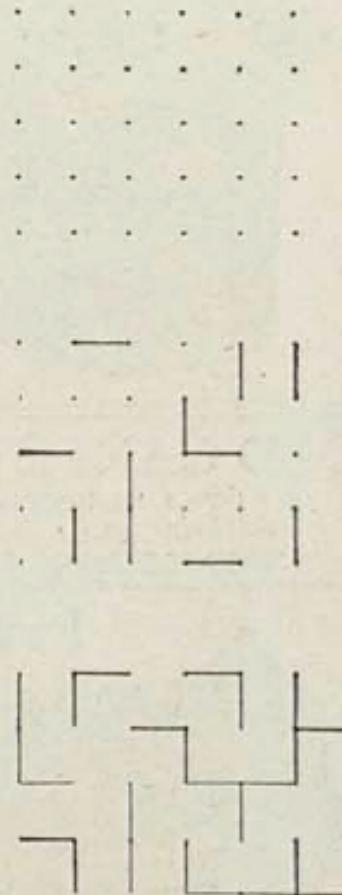
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The Dot Game

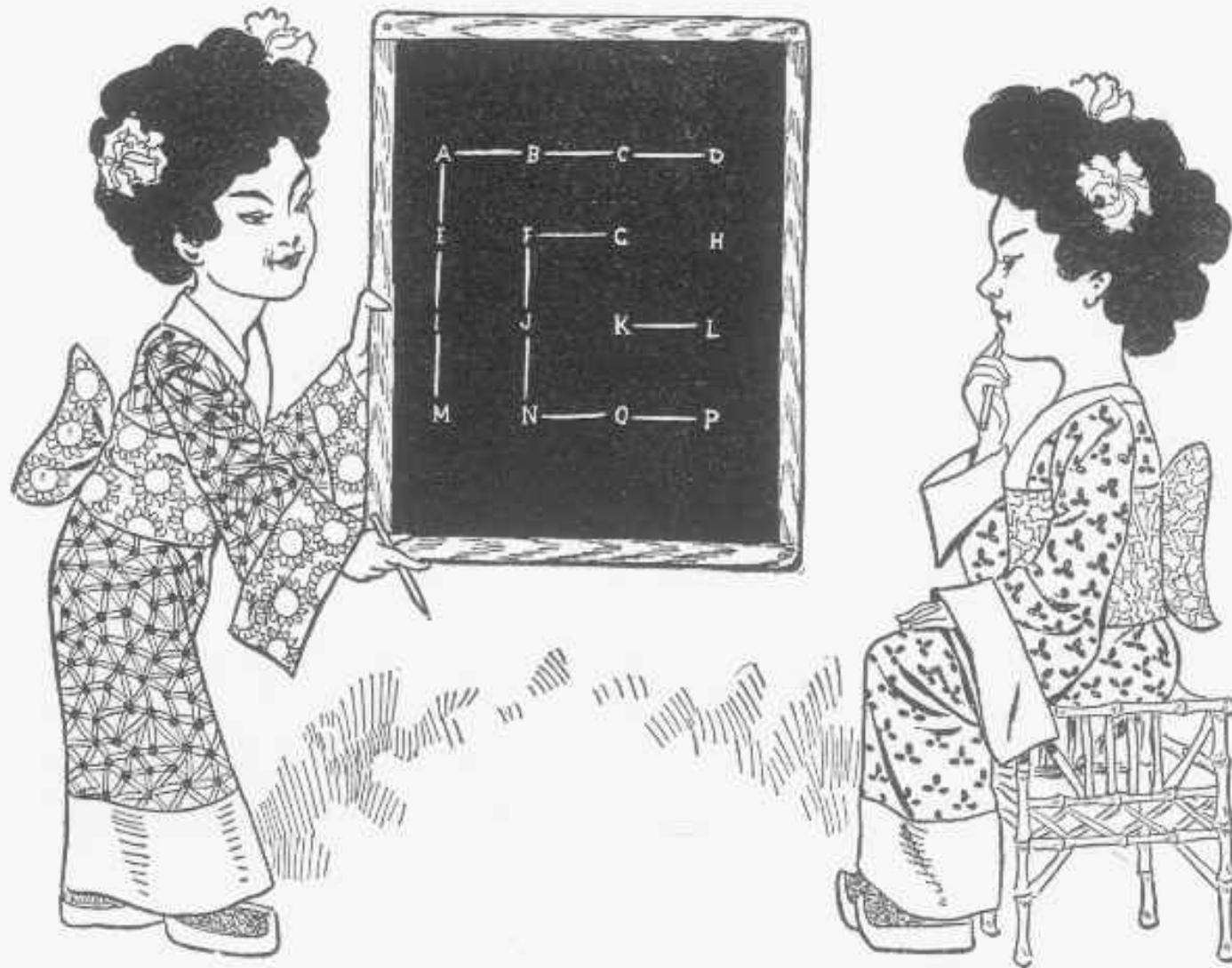
By C. E. Ganse

HAVING formed a square or oblong of dots let the children, provided with pencils, begin by joining consecutive dots, but in such a way as to prevent one another from making a complete square. For a while this is easy, but presently the places are so few where one can mark without letting the next player get a square that careful search must be taken to find a place. Finally they are all gone. The next player completes a square, and, as a reward, is allowed to join two other dots. Very likely he gets a second square, perhaps several. When he can get no more he still must join another pair of dots, and that permits the following player to score. When any one forms a square he writes in it the initial of his name, or other distinguishing mark. By this arrangement the score is readily counted.



TRY TO FORM
SQUARES

THE BOXER'S PUZZLE



What about the theory?



La Pipopipette, theory

« Sa pratique, quoique facile donne lieu à des surprises continues ; mais sa théorie n'est pas connue. Cependant nous conseillons aux nombreux amateurs, qui vont l'étudier et le propager, d'éviter avec soin la formation par l'adversaire de lignes en zigzag ou en marches d'escalier. »

« His practice, though easy, gives rise to continual surprises; but his theory is not known. However we advise the many amateurs, who will study and propagate it, to carefully avoid the formation by the opponent of zigzag lines or steps. »

Jeux scientifiques pour servir à l'Histoire, à l'Enseignement et à la Pratique du calcul et du dessin, 1889

La Pipopipette, theory

« Je n'ai pas assez pratiqué ce jeu pour pouvoir rien affirmer. Mais il me semble que, pour le second, la meilleure défense consiste à jouer les coups symétriques de ceux du premier joueur, tout du moins jusqu'à ce qu'on arrive à la fermeture des carrés. À ce moment, on devra parfois déroger à la règle ; mais le plus souvent elle assurera, je crois, le gain de la partie. »

« I have not practiced this game enough to be able to say anything. But it seems to me that for the second, the best defense is to play the symmetrical moves of the first player, at least until we reach the closing of the squares. At this point, we will sometimes have to depart from the rule; but most often it will ensure, I believe, the win of the game. »

La Pipopipette, theory

- Wilhelm Ahrens' *Altes und Neues aus der Unterhaltungsmathematik* (1918) :

La Pipopipette, theory

- Wilhelm Ahrens' Altes und Neues aus der Unterhaltungsmathematik (1918) :

Wir beginnen mit dem denkbar einfachsten Fall, dem der 2×2 Felder (Fig. 37), die wir mit den in die Figur eingetragenen Zahlen bezeichnen, während die Randlinien zwischen den Feldern in der Weise gekennzeichnet werden sollen, daß beispielsweise die Randlinie zwischen 1 und 2 als 1,2 bezeichnet wird. Heßen die Spieler A und B, wobei A der Anziehende sein mag, so kann

1	2
3	4

Fig. 37.

sich die Partie nur so abwickeln: A zieht eine Randlinie, und zwar ist es offenbar gleichgültig, welche, also etwa 1,2. Darauf zieht B 1,3, besetzt somit Feld 1 und hat daher einen weiteren Zug, zieht also etwa 2,4 und besetzt so auch Feld 2, zieht darauf schließlich 3,4 und besetzt damit auch die

Felder 3 und 4. B hat mithin alle vier Felder besetzt. Man sieht somit: Bei 2×2 Feldern siegt der Nachziehende unter Besetzung sämtlicher vier Felder. B kann sogar beinahe gar nicht anders als siegen; er müßte nämlich, wenn er absichtlich den Sieg verscherzen wollte, auf A: 1,2 geradezu antworten mit dem törichten Zug: 3,4, worauf A durch die Züge 1,3 und 2,4 alle vier Felder erobern würde.

La Pipopipette, theory

- Wilhelm Ahrens' Altes und Neues aus der Unterhaltungsmathematik (1918) :

Felder besetzen. Nachdem *B* nämlich 3,5 gezogen hat, handelt es sich nur noch um das Quadrat der 2×2 Felder, und es ist daher bei der völligen Symmetrie dieser Restfigur gleichgültig, welchen der Züge wir für *A* annahmen. Sagen wir, daß er 1,2 gezogen hat, so zieht *B* etwa der Reihe nach 1,3; 2,4; 3,4, wie im Falle der Fig. 37. Damit ist endgültig gezeigt, daß es für *A* keinen Eröffnungszug gibt, der seine Niederlage abwenden könnte, und wir dürfen somit sagen: Bei dem Spielrahmen der Fig. 38 (fünf Felder) siegt der Nachziehende.

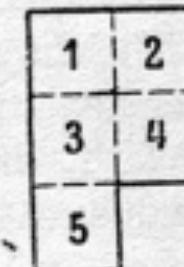


Fig. 38.

La Pipopipette, theory

- Wilhelm Ahrens' Altes und Neues aus der Unterhaltungsmathematik (1918) :

Als letztes Beispiel wollen wir ein Gebiet von 3×3 Feldern (Fig. 41) studieren. *A* eröffnet mit 4,5 und gewinnt, wie wir alsbald erkennen werden. Wenn *B* als ersten Zug etwa 1,2 tut, so *A* der Reihe nach: 1,4 (Besetzung von 1); 4,7 (Besetzung von 4); 7,8 (Besetzung von 7). Es bleibt dann nur noch ein Gebiet von 2×3 Feldern; wir haben somit den Fall der Fig. 39 und, da *A* noch einen Zug zu tun hat, so siegt er unbedingt. — Im wesentlichen dieselben Verhältnisse haben wir, wenn der erste Zug von *B*, statt 1,2, lautet: 1,4 oder 4,7 oder 7,8. Damit sind denn für den ersten Zug von *B* alle die Möglichkeiten erledigt, die dem linken Streifen, der von den Feldern 1, 4, 7 gebildet wird, angehören, und wir haben in Anbetracht der Symmetrie nur noch folgende wesentlich verschiedene Fälle für den ersten Zug von *B* zu berücksichtigen:

- I. 2,3;
- II. 3,6;
- III. 2,5;
- IV. 5,6.

1	2	3
4	5	6
7	8	9

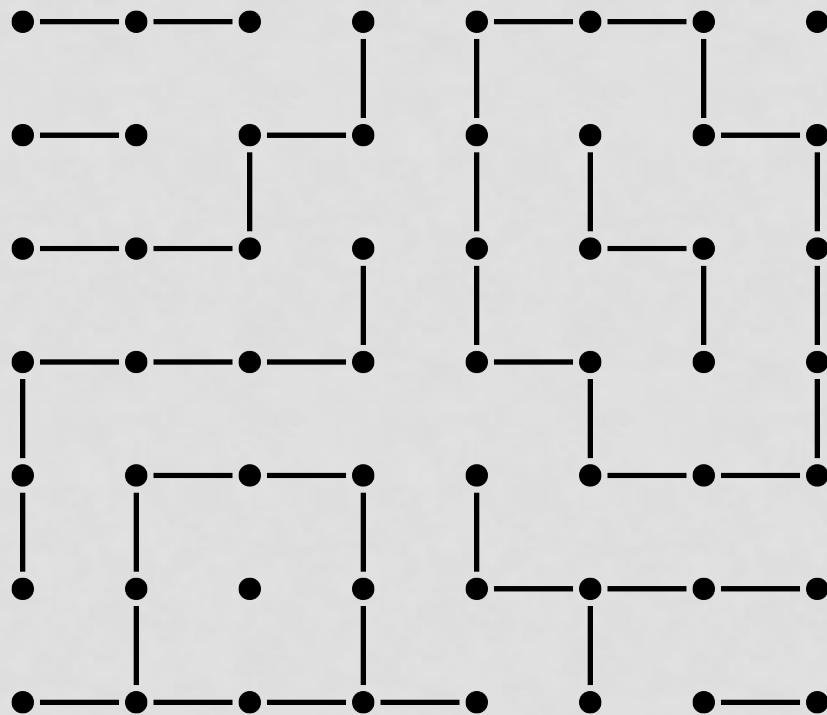
Fig. 41.

La Pipopipette, theory

- John Holladay: “A Note on the Game of Dots”,
American Mathematical Monthly, 1966 :

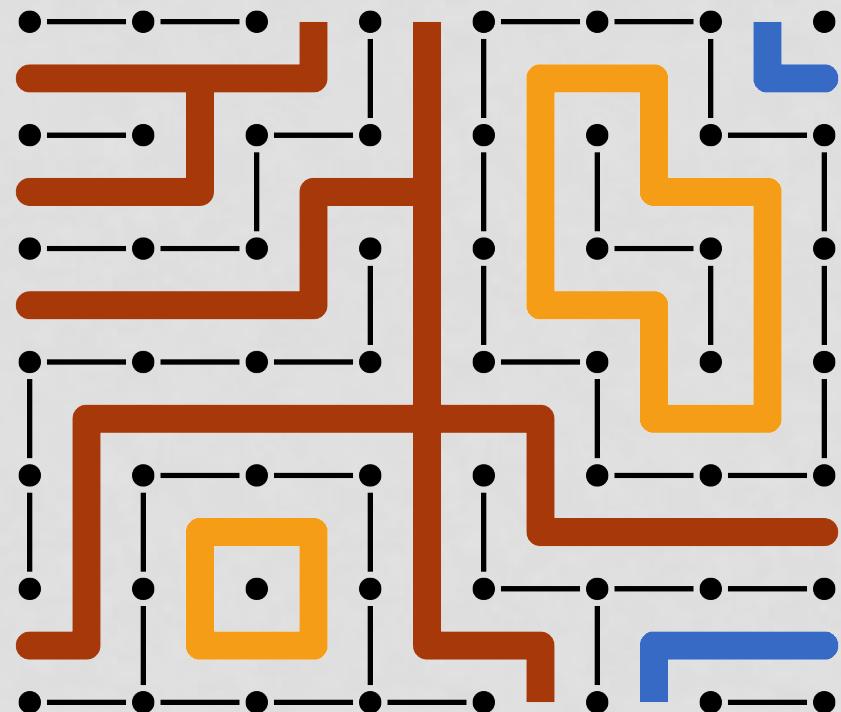
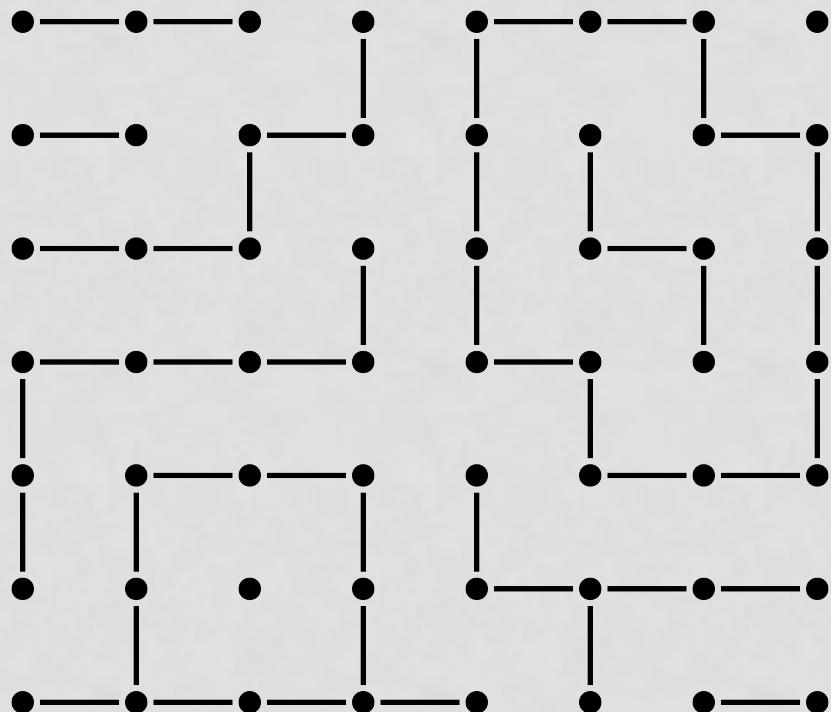
La Pipopipette, theory

- John Holladay: “A Note on the Game of Dots”,
American Mathematical Monthly, 1966 :



La Pipopipette, theory

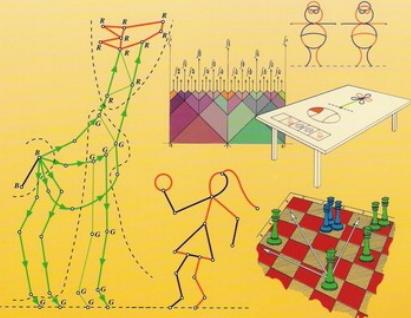
- John Holladay: “A Note on the Game of Dots”,
American Mathematical Monthly, 1966 :



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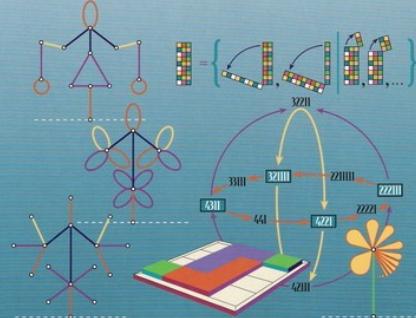
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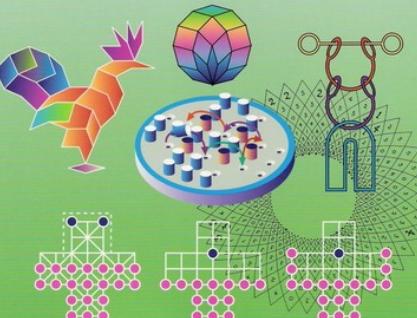
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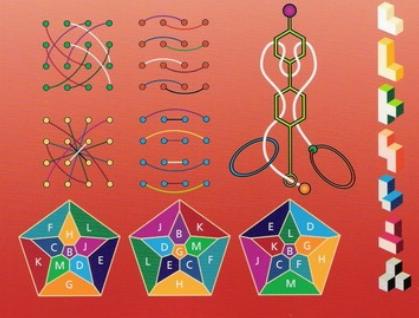
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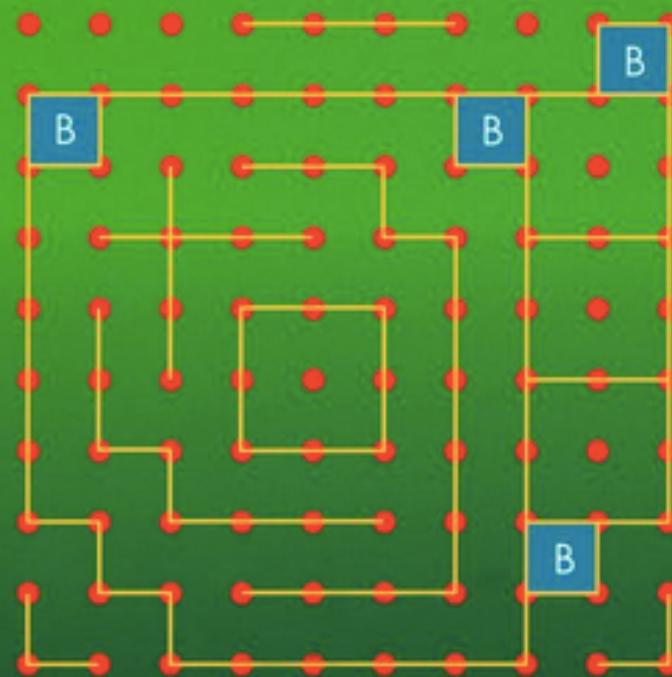
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(1966-1982)

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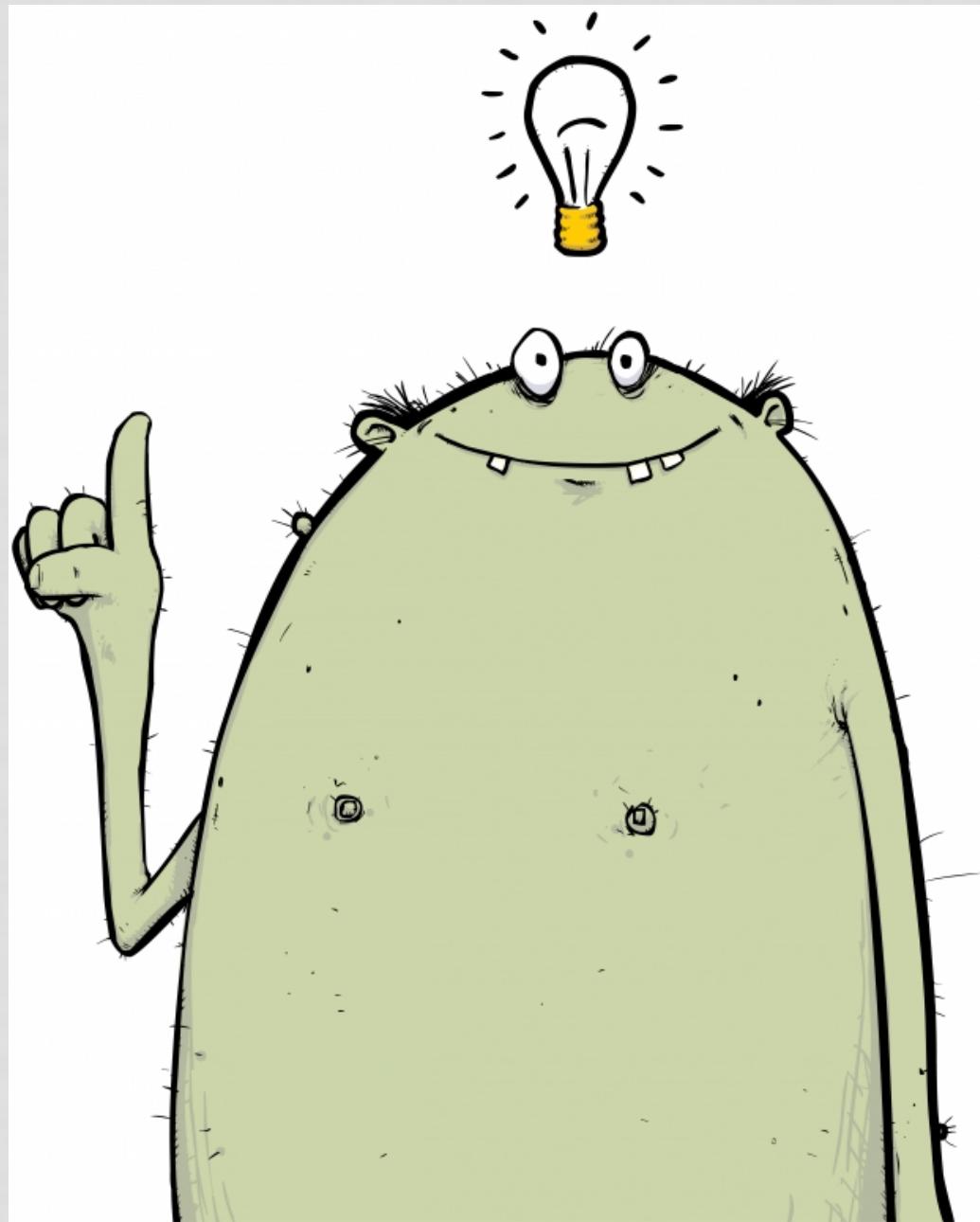
THE DOTS AND BOXES GAME

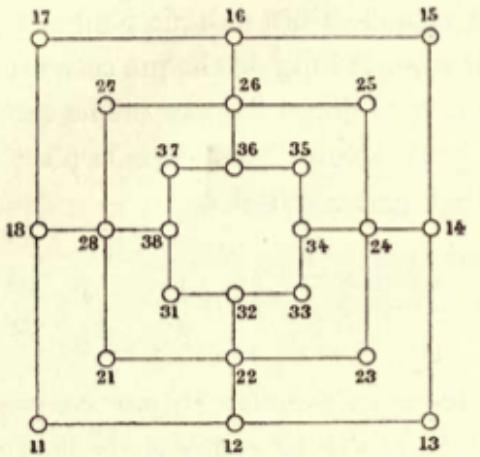
SOPHISTICATED CHILD'S PLAY



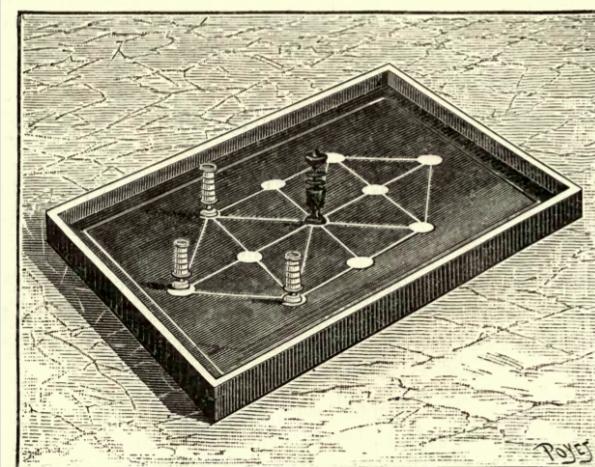
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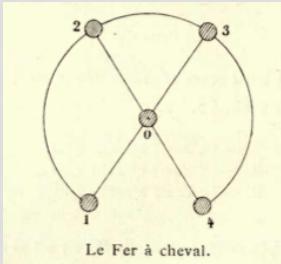


Notation de la Marelle triple.



Le nouveau jeu militaire.

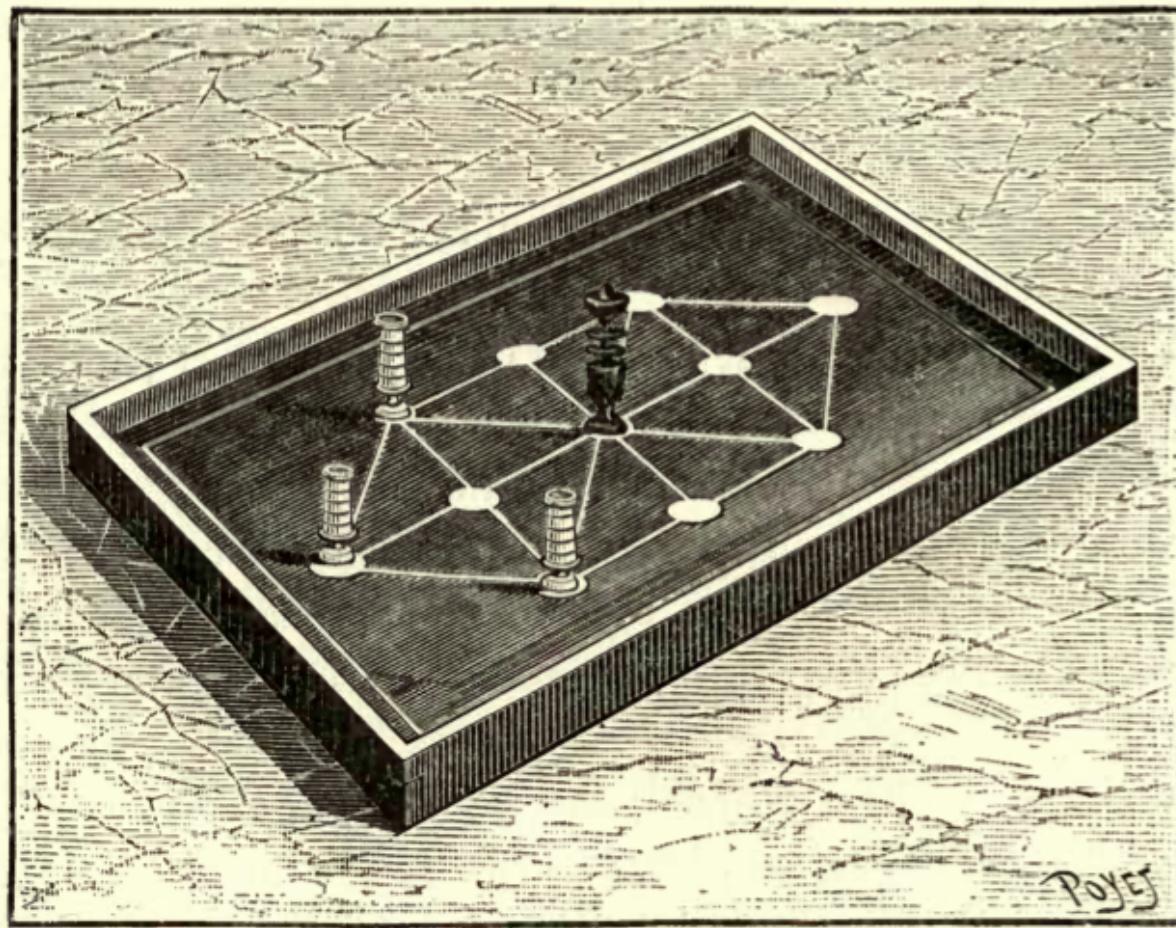
Other “Jeux de combinaisons” at the end of the 19th century



Le Fer à cheval.

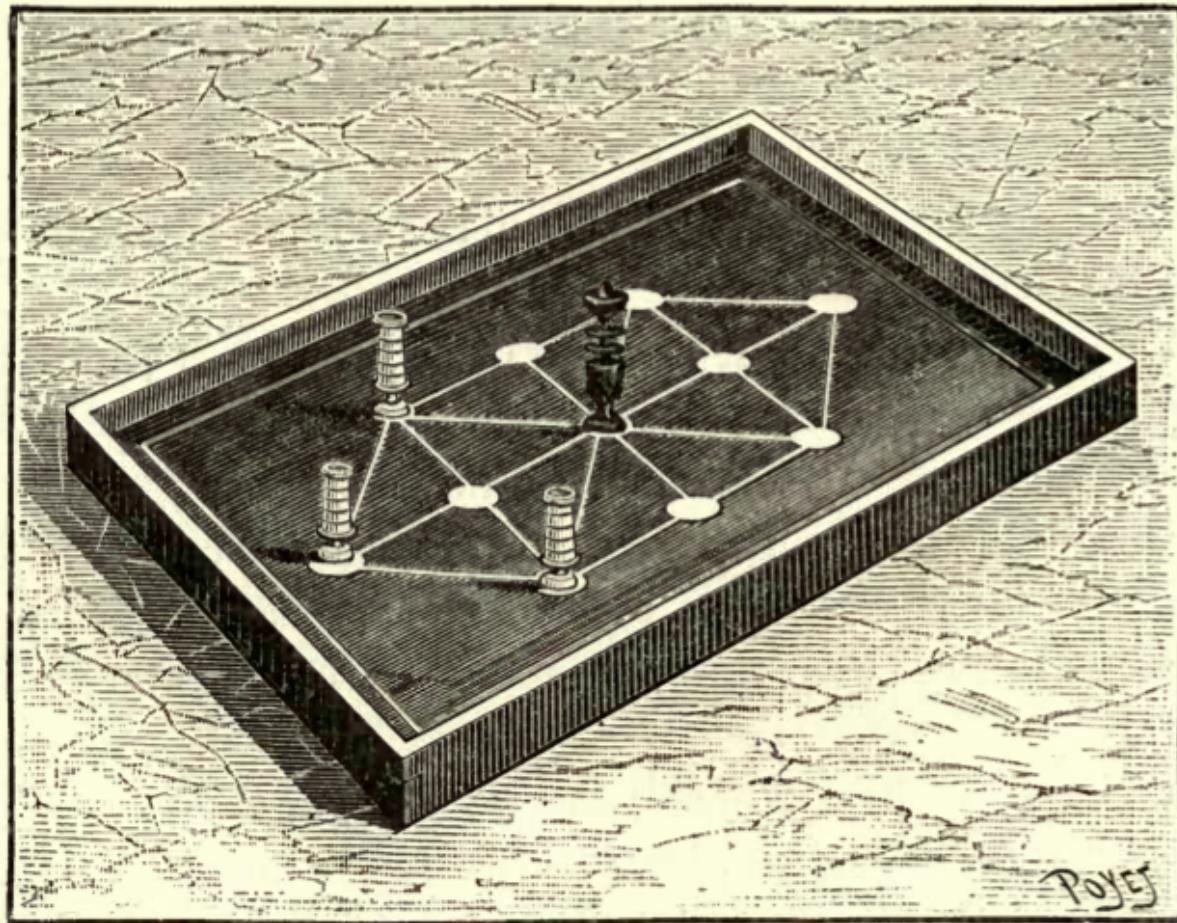


Le jeu militaire (Vol. 3, 1893)



Le nouveau jeu militaire.

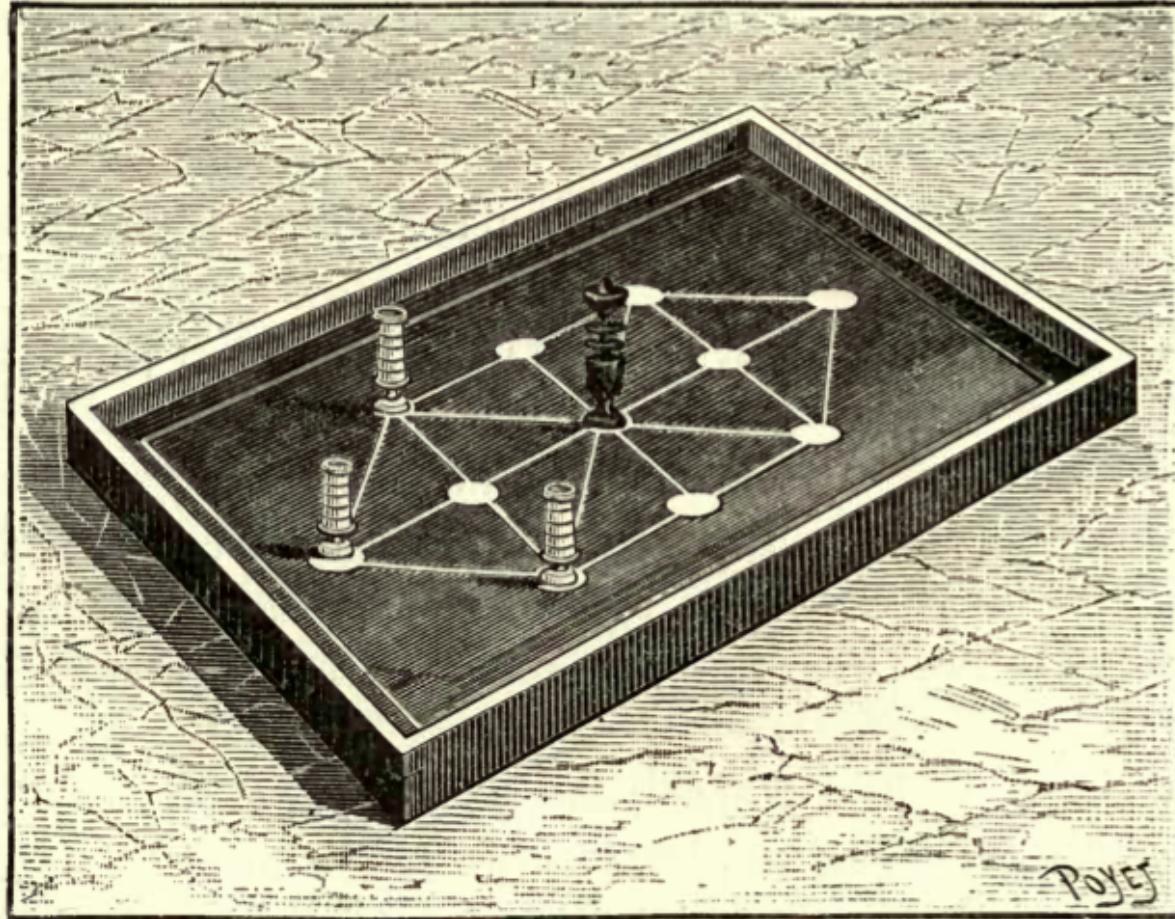
Le jeu militaire



Le nouveau jeu militaire.

« Less difficult than chess, the 'Jeu militaire' is among the most instructive ones, and deserves to be recommended as a very useful distraction for officers and non-commissioned officer. »

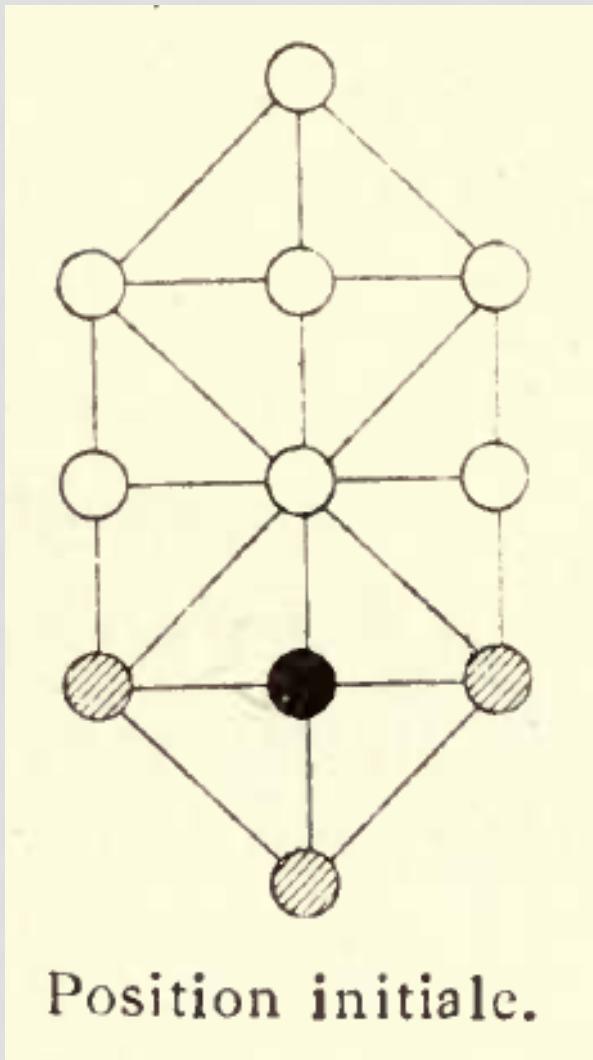
Le jeu militaire



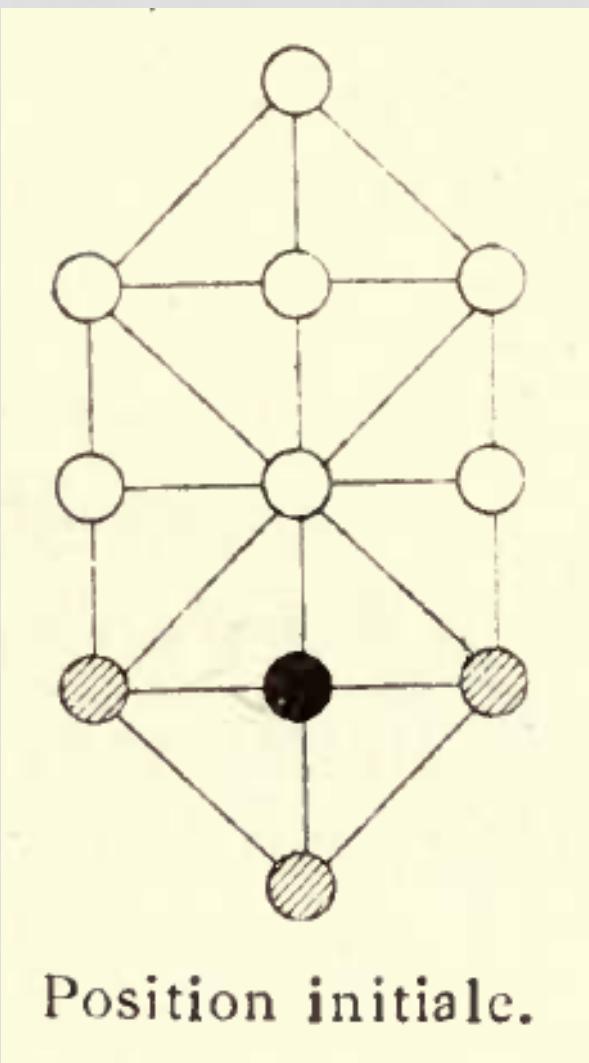
Le nouveau jeu militaire.

« Bonuses of hundred 'francs' are offered by the inventor to the persons who will win as many games as himself, and bonuses of thousand 'francs' to those who would win more than half of them. »

Le jeu militaire: rules

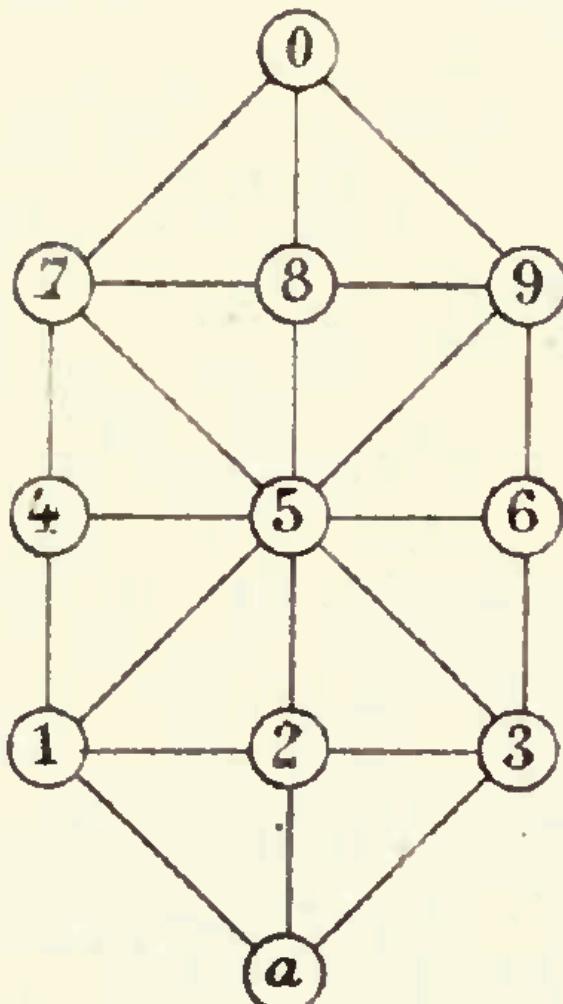


Le jeu militaire: rules

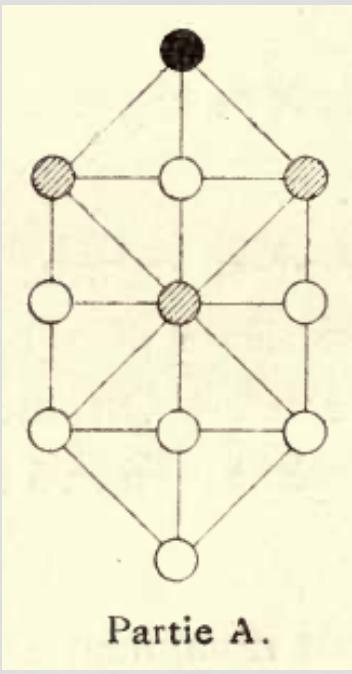


- The army = black token (J1)
- The towers = white tokens (J2)
- Aim of the army: not be blocked by the towers.
- Aim of the towers: block the army.
- Moves : the army(J1) can move one step in any direction. Towers can move only forward and laterally (one tower at a time).

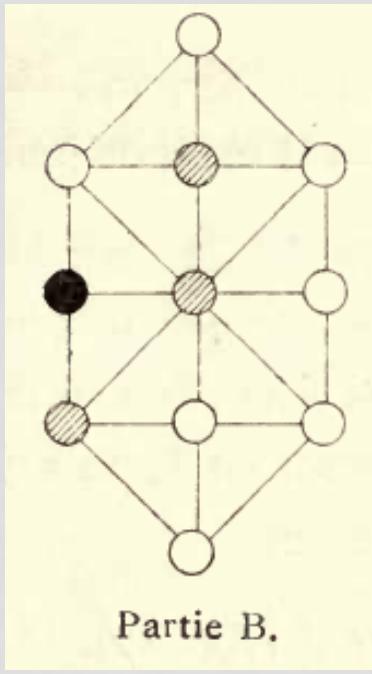
Le jeu militaire



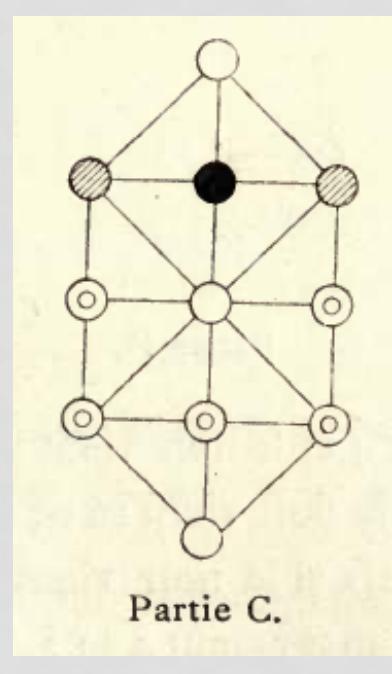
- Positions are numbered.
- Initial position: black on 2 and whites on 1, a and 3.
- Notation : 1a3 – 2



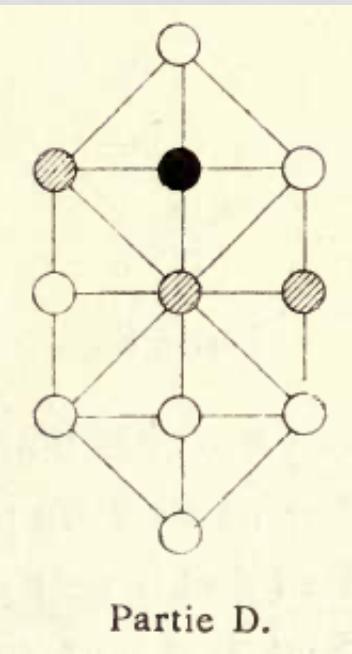
Partie A.



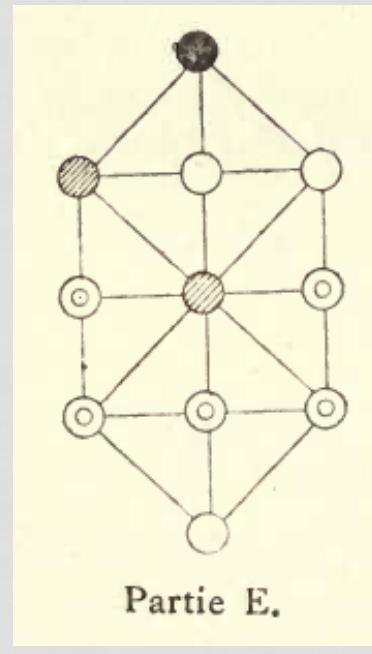
Partie B.



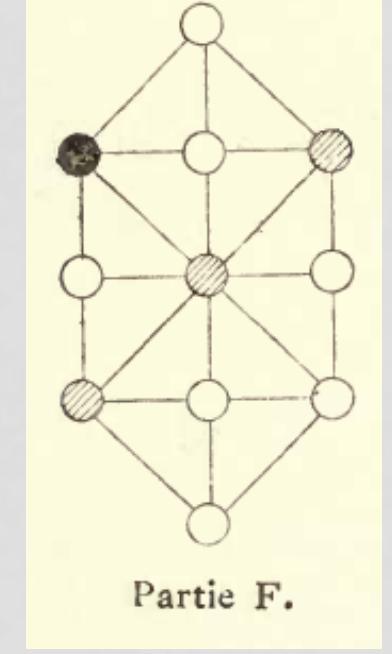
Partie C.



Partie D.

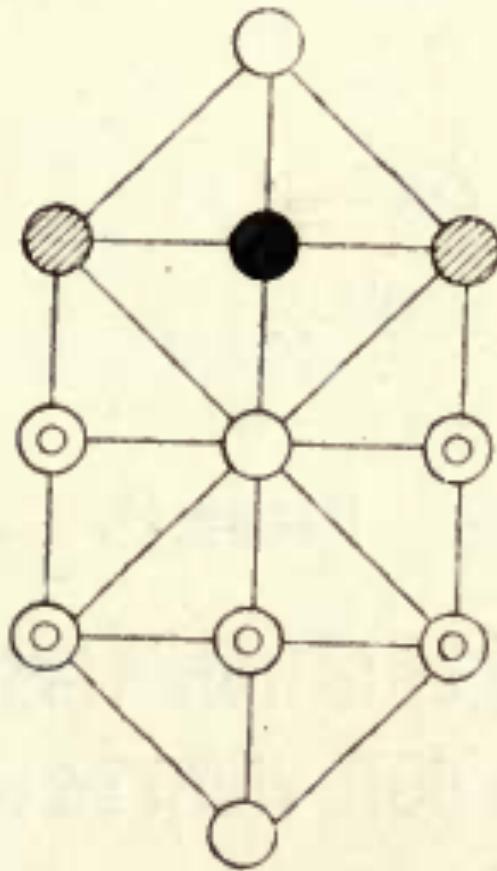


Partie E.



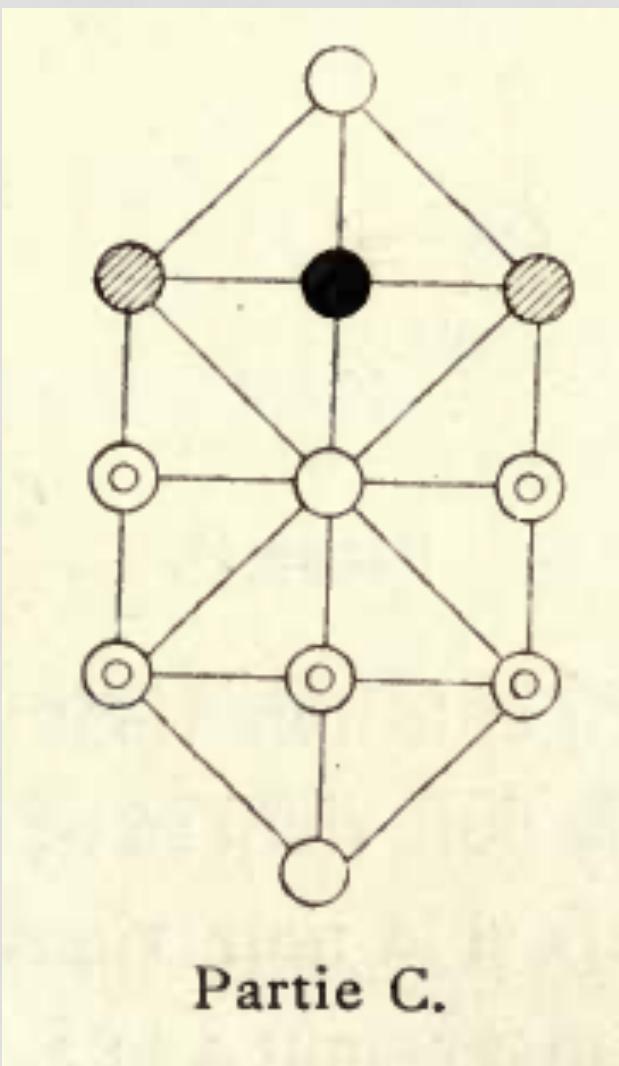
Partie F.

Le jeu militaire: exemple 1



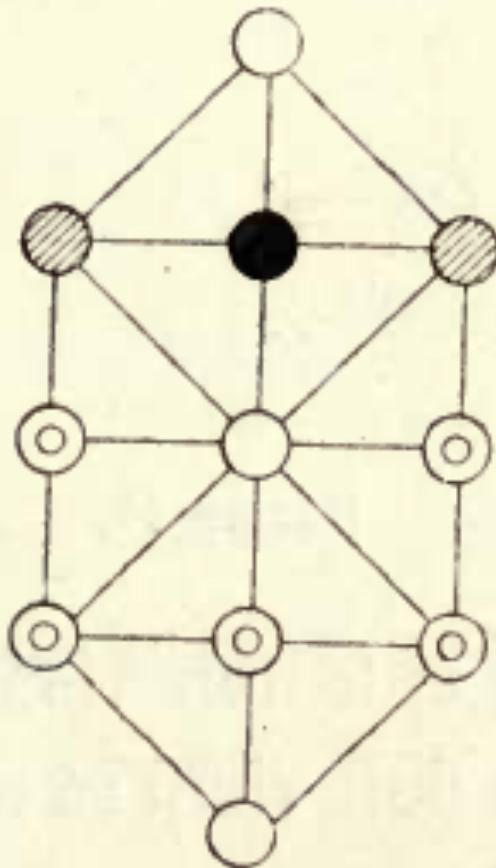
Partie C.

Le jeu militaire: exemple 1



- White wins in two moves.
- (The 3rd tower can be anywhere on the spotted positions.)

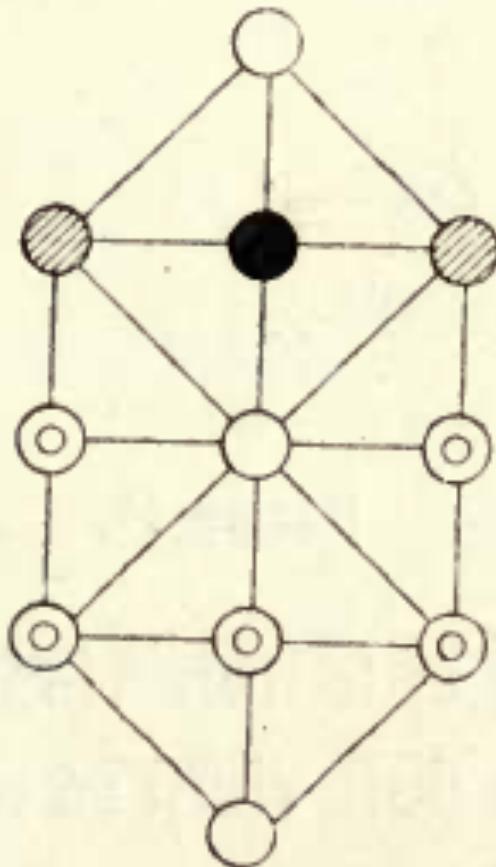
Le jeu militaire: exemple 1



Partie C.

- White wins in two moves.
- (The 3rd tower can be anywhere on the spotted positions.)
 - White plays the 3rd tower on the center (5)
 - Black plays on 0.
 - White moves the tower from 5 to 8.

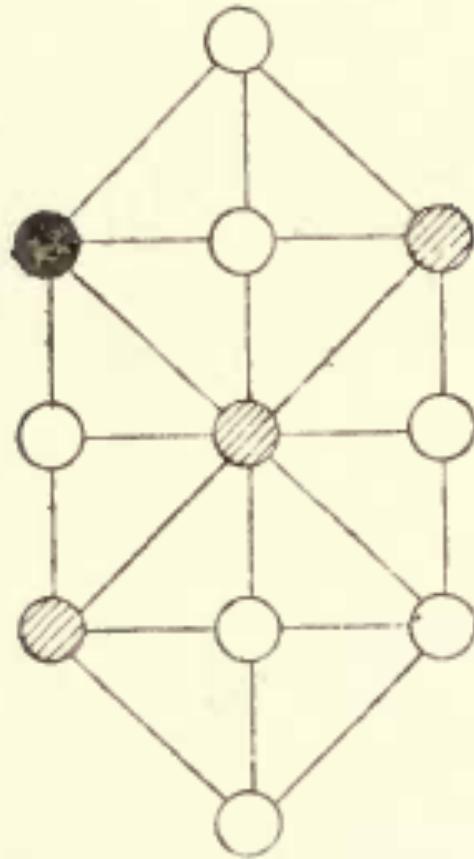
Le jeu militaire: exemple 1



Partie C.

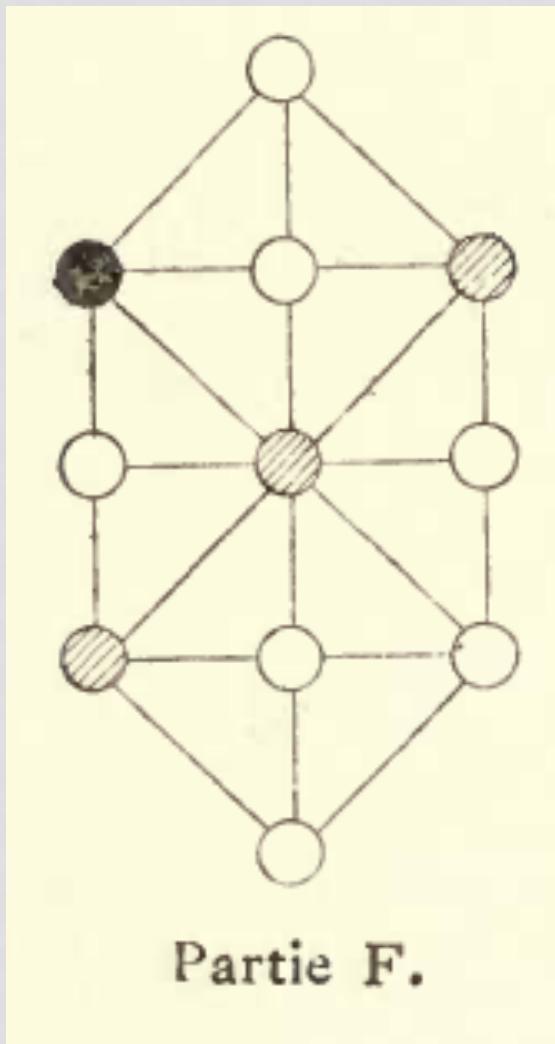
- White wins in two moves.
 - (The 3rd tower can be anywhere on the spotted positions.)
 - White plays the 3rd tower on the center (5)
 - Black plays on 0.
 - White moves the tower from 5 to 8.
- Notation: 179 - 8

Le jeu militaire: exemple 2



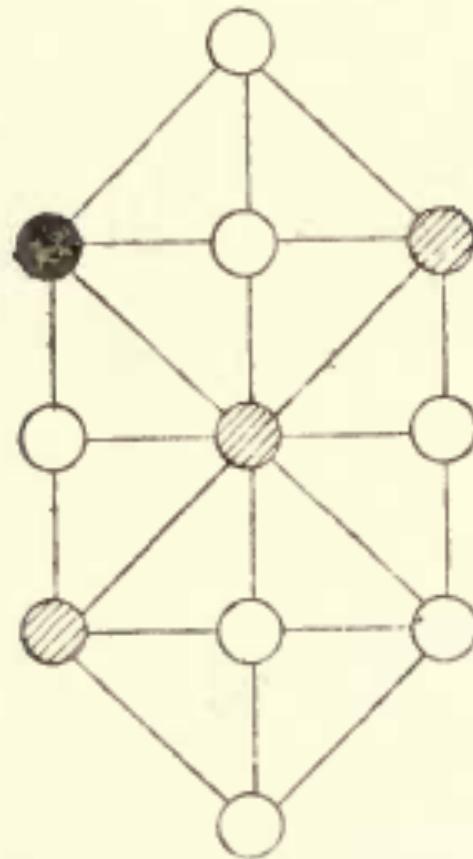
Partie F.

Le jeu militaire: example 2



- White wins in three or four moves.

Le jeu militaire: example 2

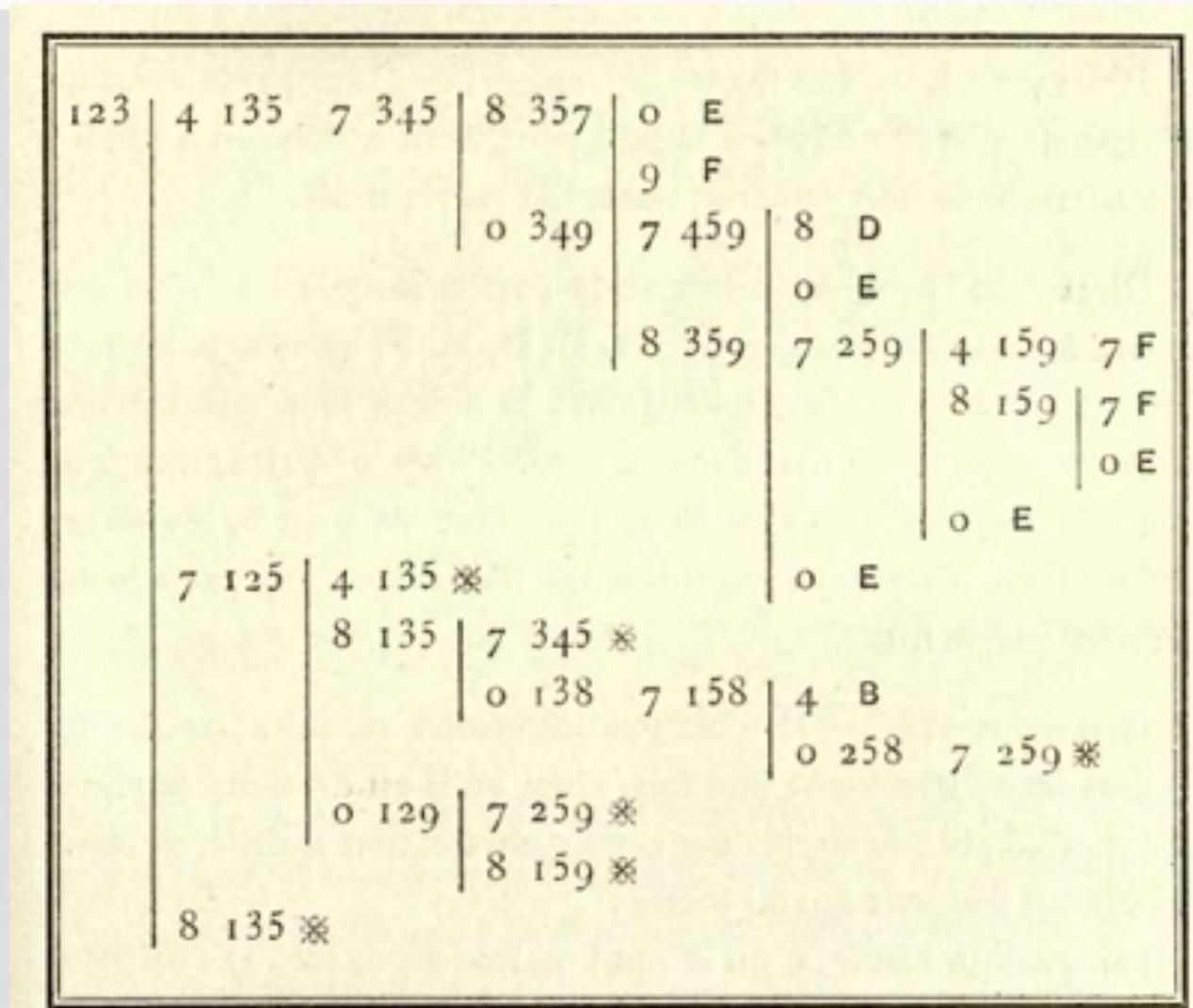


Partie F.

- White wins in three or four moves.

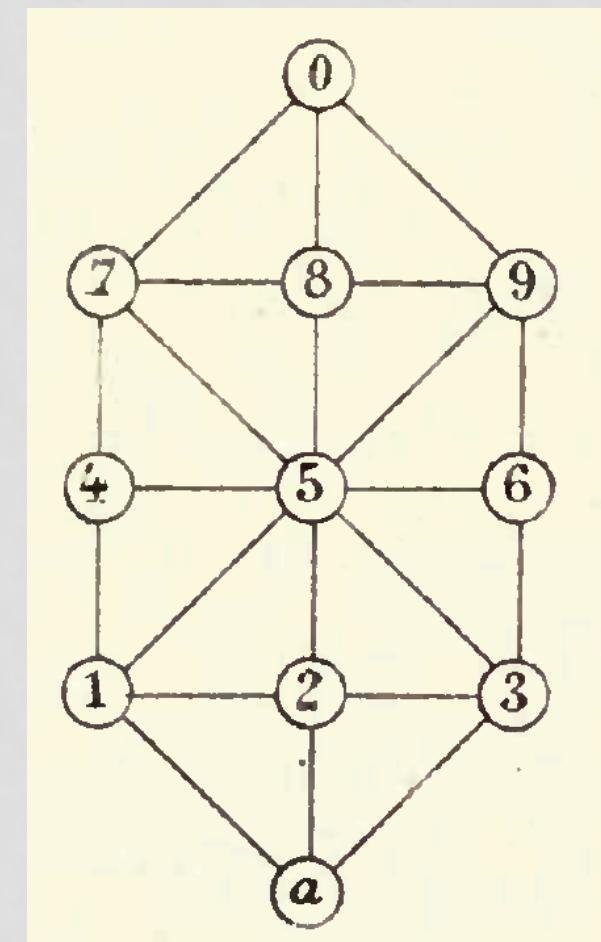
159	7	459		8	579	0	789	»	»
0	479	8	579	0	789				

Le jeu militaire: “the game tree”



Le jeu militaire: “the game tree”

1 2 3	4 1 3 5	7 3 4 5	8 3 5 7	0 E
			0 3 4 9	9 F
			7 4 5 9	8 D
			8 3 5 9	0 E
			7 2 5 9	4 1 5 9 7 F
				8 1 5 9 7 F
				0 E
7 1 2 5	4 1 3 5 *			
	8 1 3 5	7 3 4 5 *		
		0 1 3 8 7 1 5 8	4 B	
			0 2 5 8 7 2 5 9 *	
	0 1 2 9	7 2 5 9 *		
		8 1 5 9 *		
8 1 3 5 *				

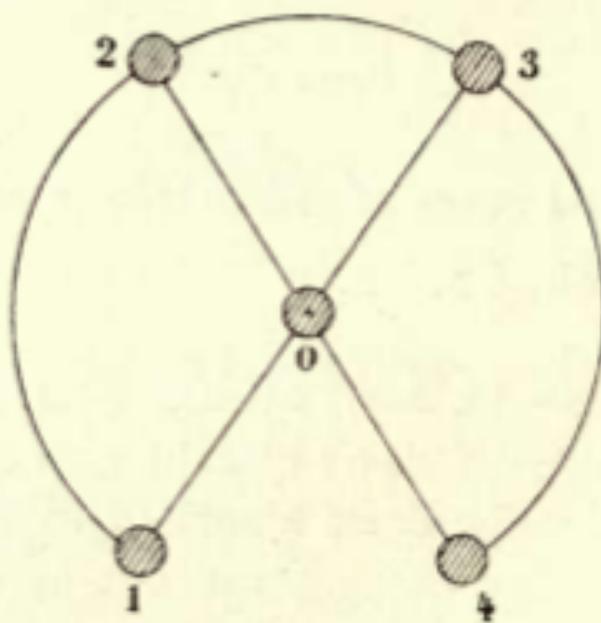


Le jeu militaire: “the game tree”

1 2 3	4 1 3 5	7 3 4 5	8 3 5 7	o E
			9 F	
	o 3 4 9	7 4 5 9	8 D	
			o E	
	8 3 5 9	7 2 5 9	4 1 5 9	7 F
			8 1 5 9	7 F
				o E
7 1 2 5	4 1 3 5 *		o E	
	8 1 3 5	7 3 4 5 *		
		o 1 3 8	7 1 5 8	4 B
			o 2 5 8	7 2 5 9 *
	o 1 2 9	7 2 5 9 *		
		8 1 5 9 *		
8 1 3 5 *				

« In few hours we can master the game, and win the prize of hundred ‘francs’ offered by the leaflet as many times as we want. But it’s about finding the address of the banker; this problem is more difficult than the previous one. »

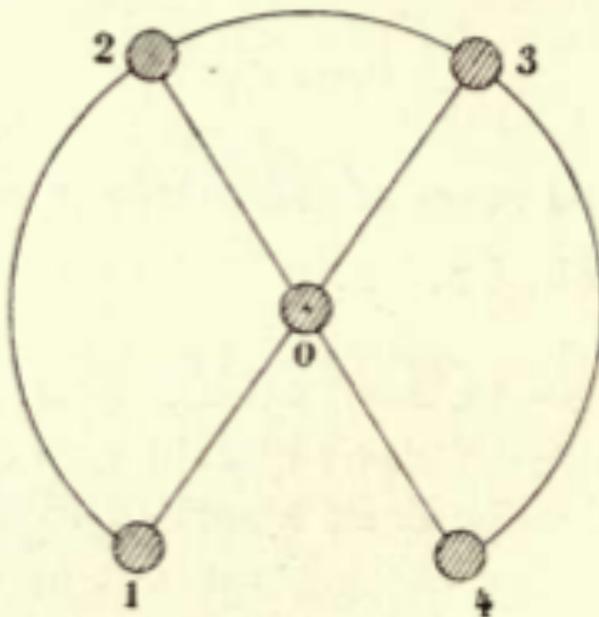
Le fer à cheval (the horseshoe)



Le Fer à cheval.

The Horseshoe

Le fer à cheval (the horseshoe)



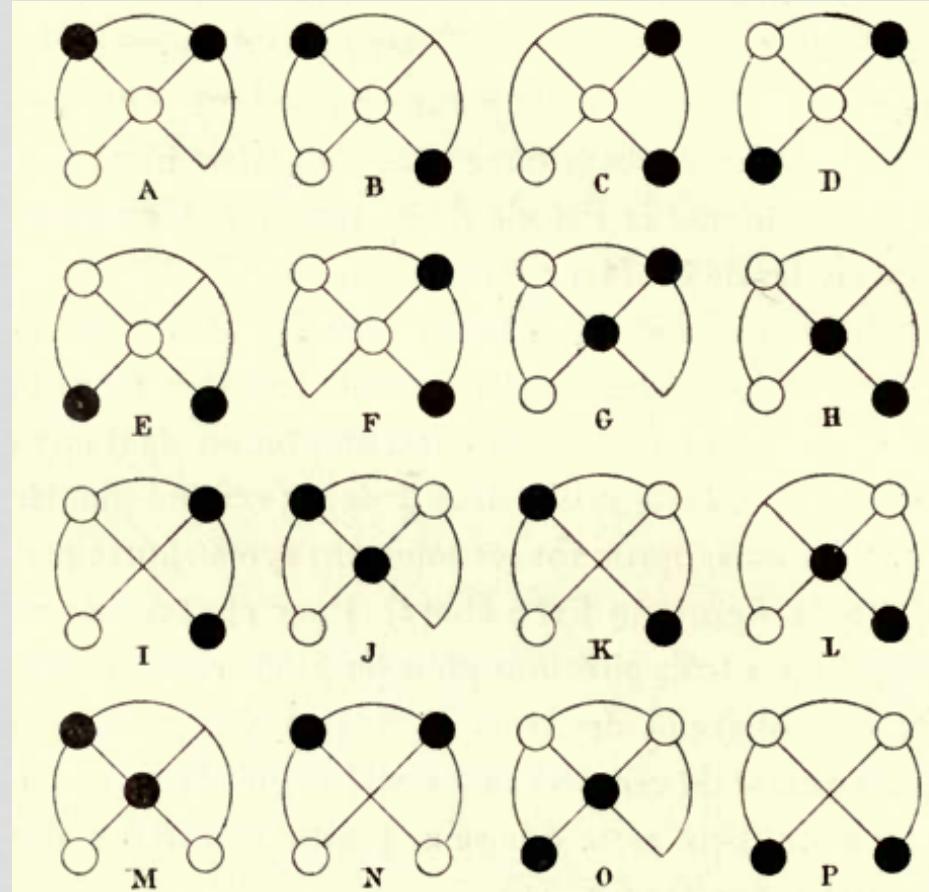
Le Fer à cheval.

The Horseshoe

« The game is played by two players; each player has different colored tokens, white and black, for example. The two players alternately place their two tokens on an empty square, then, in turn, drag one of their pieces onto a neighboring position. The game is lost by the player who no longer moves any of his tokens and who has been blocked. »

Le fer à cheval (the horseshoe)

Sixteen games are possible
(each named by a letter):

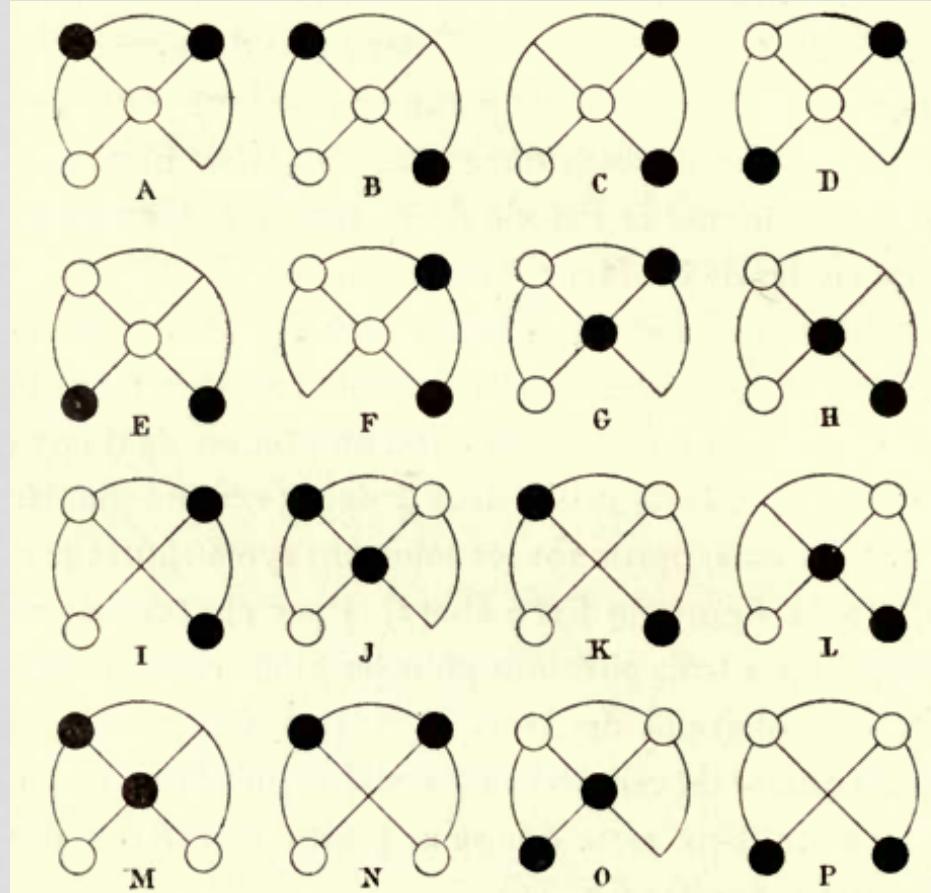


Les seize positions dans le Fer à cheval.

Le fer à cheval (the horseshoe)

Sixteen games are possible
(each named by a letter):

- in 1 game Black wins: G
- in 2 games White wins: C and I
- the other 13 games are draws.



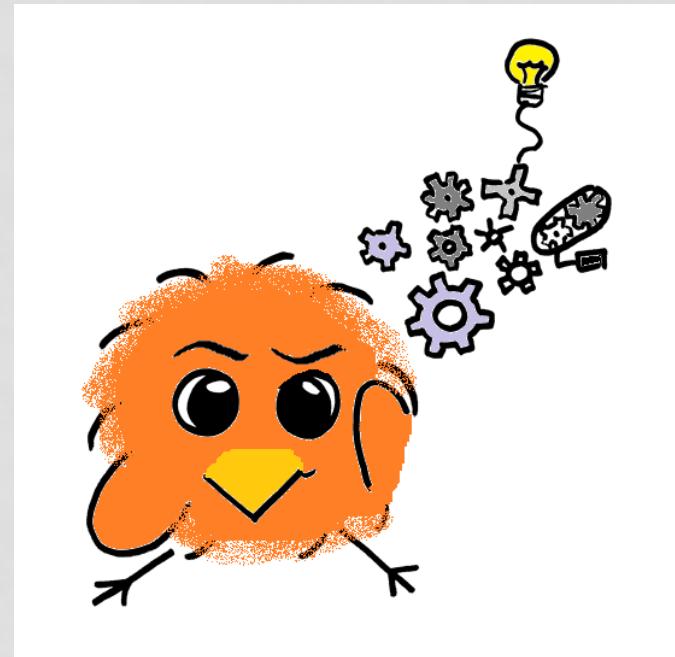
Les seize positions dans le Fer à cheval.

Conclusion

- At the beginning of the 20th century the term “jeu de combinaison” has a proper definition (similar to “combinatorial game”).
- This part of the (hi)story is not very well known and is an alternative to the common Bouton-Sprague/Grundy-Conway timeline for CGT history.
- But, it is still difficult – as for games in general – to trace back the origins of a mathematical analyze of a game...

Thank you for your attention!

Any questions?



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