Which Rules for Mu Torere?

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Abstract. Mu Torere is a traditional board game played by the Maori people of New Zealand. It has simple rules, low complexity and has been fully analysed, but surprisingly is often described with incorrect rules in the literature. This paper compares the various known rulesets for Mu Torere to investigate which provides the most interesting game, as the first step in a more thorough analysis of this game.

Keywords: Mu Torere · Board Game · Game AI · Game Analysis.

1 Introduction

Mu Torere is a traditional board game played by the Maori people of New Zealand [3]. The game is played by two players on the 9-point board shown in Figure 1 (left). Both players start with four pieces of their colour initially set as shown (Figure 1, centre).

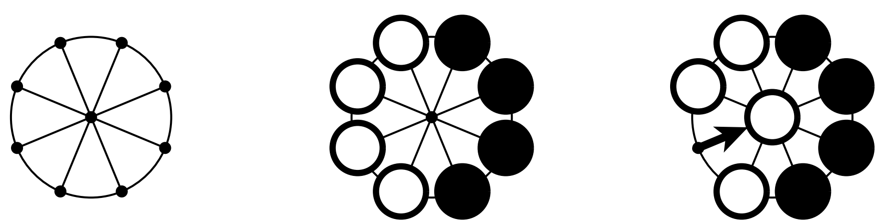

Fig. 1. The Mu Torere board (left), starting position (centre) and winning move (right).

Starting with White, players alternate moving a piece of their colour, according to the following basic rules:

1. The mover must move a piece of their colour to the adjacent empty point.
2. A player wins if the opponent cannot move on their turn.

However, a moment's analysis reveals a crippling problem with these basic rules. White immediately has a winning move from the starting position (Figure 1, right), so the game is typically played with additional restrictions on movement. The following five restrictions are found in the literature, giving five variant rulesets:

- **Ruleset A:** No restriction.
- **Ruleset B:** White cannot win on the first move.
- **Ruleset C:** The piece being moved must be adjacent to an opponent's piece.
- **Ruleset D:** The mover cannot win during the first two rounds.
- **Ruleset E:** The piece being moved must be adjacent to an opponent's piece if it is being moved to the centre point.

Unfortunately, it is not clear from the written accounts which of these rulesets is typically used in practice. This paper presents an analysis of the five rulesets to investigate which produces the most interesting game.

2 Ruleset A: No Restriction

Ruleset A represents the basic unrestricted form of the game. It is typically not found in historical descriptions of the game, but can be found in modern sources that oversimplify the description of the game.[1]

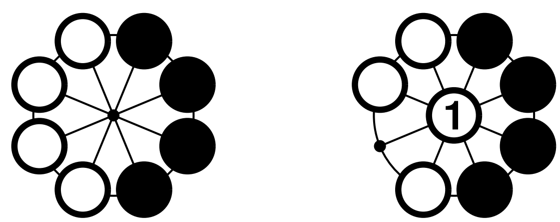

Fig. 2. Trivial win in one move for White using Ruleset A.

This ruleset allows White to win on the first move, as shown in Figure 2. This ruleset should not be used.

3 Ruleset B: No Win On First Move

Ruleset B circumvents this win-in-1 problem by explicitly forbidding White to win on the first move. This ruleset is found in historical accounts including [6].

However, this ruleset does not solve the trivial win problem but rather just delays it slightly. Black can force a win every game as shown in Figure 3. This ruleset should not be used.

[1] See for example: http://gamescrafters.berkeley.edu/games.php?game=mutorere

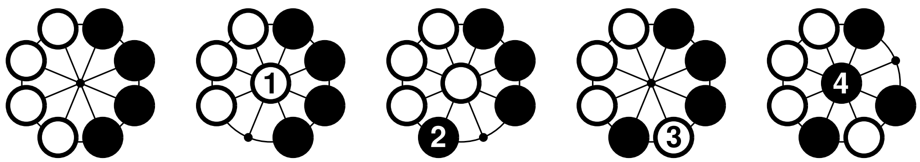

Fig. 3. Trivial win in four moves for Black using Ruleset B.

4 Ruleset C: Move If Adjacent To Enemy

Ruleset C circumvents the trivial win problem of Rulesets A and B indirectly by forbidding moves that would allow the win-in-1 and win-in-4 cases. This ruleset is found in modern descriptions of the game.[2]

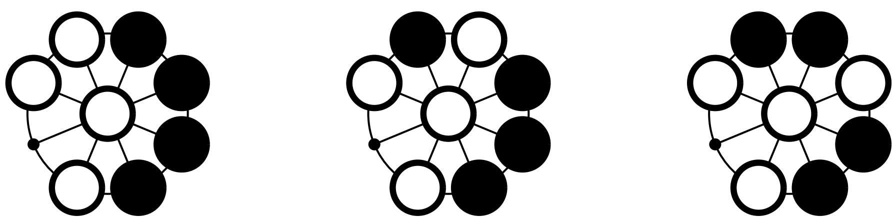

Fig. 4. The three basic winning patterns for White.

Unfortunately, this restriction is too successful in its aim and prevents *any* winning position from occurring. Figure 4 shows the three basic winning patterns from White's perspective; the restriction that pieces can only move if they are adjacent to an enemy piece does not allow any of theses positions to occur (for either player) so every game is guaranteed to become a perpetual cycle. This ruleset should not be used.

5 Ruleset D: No Win On First Two Rounds

Ruleset D is the most prevalent ruleset throughout the literature, both from historical sources [1], [2], [3], [4] and from mathematical analyses of the game [10], [11]. This ruleset simply forbids either player winning on their first two moves.

This rule is often described in more complicated terms, e.g. "For each player's first two moves, a stone can be moved from an outer node to the center only if it is adjacent to an opponent's stone" [10, p.384], but this is functionally equivalent to: "The mover cannot win during the first two rounds."

[2] See for example: https://cpb-ap-se2.wpmucdn.com/blogs.auckland.ac.nz/dist/7/67/files/2018/04/mu_torere_board-1b2pdv1.pdf

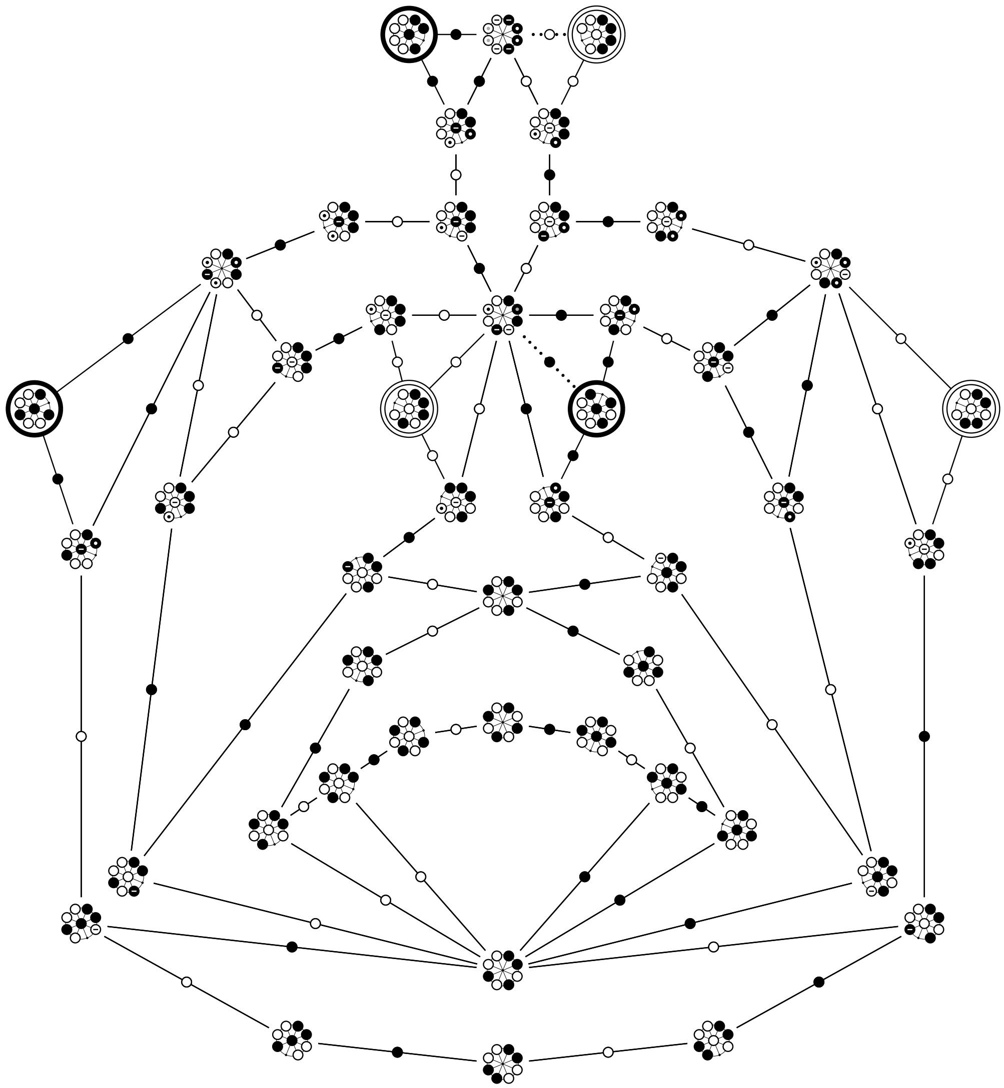

Fig. 5. Complete game graph for Ruleset D.

Figure 5 shows a full game graph expansion of Mu Torere played with Ruleset D. The format is based on Straffin's beautiful analysis of the game ([10] and [11]), except that the actual board positions are shown at each node for convenience and the game-theoretic value of each possible move is shown, from the perspective of each piece's owner, as follows:

- **Win ⊙:** The mover can force a win with this move.
- **Loss ⊖:** The opponent can force a win if the mover makes this move.
- **Draw (no marking):** Move that leads to an infinite loop with optimal play.

The graph shows the 46 distinct positions that can occur during play, with each node representing all rotations and reflections of that position, and each edge between nodes representing all moves by the player indicated that produce a transition from one state to the other. The graph has six terminal positions representing winning or losing positions, each indicated by a surrounding circle in the winner's colour. The graph essentially provides a set of instructions for which moves to play – and not to play – from each position.

The initial state is shown at the centre top; note that this position and the position immediately below are effectively superpositions of these positions during the first two rounds (when winning moves – dotted – are forbidden) and subsequent later (when the dotted moves are allowed). Note that three of the symbols indicating moves as game-theoretic wins ⊙ and losses ⊖ in these positions are greyed; this indicates that these values only apply after the first two rounds.

6 Ruleset E: Move To Centre If Adjacent To Enemy

This variant is described by Bell [4], [5] – who hedges his bets by listing both variants D and E in [4] – and used by Jelliss in his excellent mathematical analysis of the game [7]. Reed's visual description of the game's rules through example [9] is compatible with both Rulesets D and E.

Figure 6 shows a full game graph expansion of Mu Torere played with Ruleset E. Jelliss [7] uses a more compact representation in his analysis that yields 26 distinct positions (each with a colour inversion flag) so that positions can conveniently be labelled by letters of the alphabet. However, Straffin's graph layout is used here for readability.

The game produced by Ruleset E is similar to that produced by Ruleset D except that more moves are forbidden. This can be seen in the game graph for Ruleset E, which is similar to the game graph for Ruleset D except that six winning moves shown in graph D are absent in graph E, and fewer moves are labelled as winning ⊙ or losing ⊖, hence more moves are drawish.

7 Which Ruleset: D or E?

It is clear that Rulesets A, B and C can be discarded due to producing: (A) a trivial win, (B) a trivial loss, and (C) the impossibility of a result. Rulesets D and E both produce relatively well-behaved and interesting games for the low complexity involved. Can we identify either of these superior rulesets as more potentially interesting than the other?

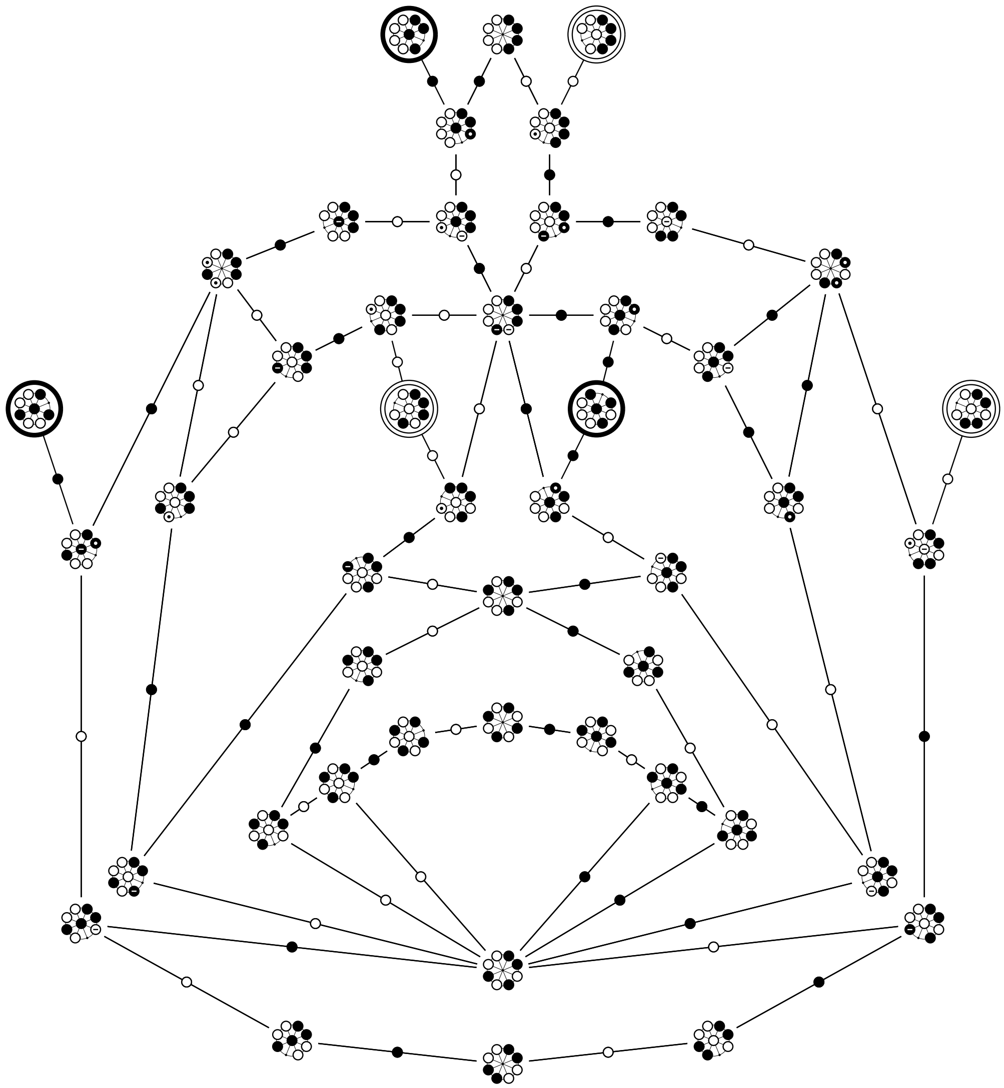

Fig. 6. Complete game graph for Ruleset E.

7.1 Drawishness

Mu Torere is a game of low complexity and is drawish in nature – no experienced player should ever lose a game – but it is still of interest to human players. It allows distinct skill levels and a skilled player will generally beat a beginner, and there is anecdotal evidence that (skilled) local Maori players have consistently beaten (unskilled) foreign opponents [2].

In order to make the game as interesting as possible, its rules should discourage draws and encourage a result as often as possible (assuming that both players are not experts). Table 1 gives some insight into the nature of Rulesets D and E by comparing the ratio of win, draw and loss game-theoretic values across positions and transitions that may occur during play.

Table 1. Distribution of Game-Theoretic Values Across Game Graphs D and E.

		Win	**Draw**	**Loss**
Ruleset D	Positions	6	25	15
	Transitions	24	74	32
Ruleset E	Positions	5	31	10
	Transitions	14	94	16

Ruleset D appears to be less drawish in nature. It only has fewer drawish positions than Ruleset E (25 compared to 31), more winning positions (6 compared to 5) and more losing positions (15 compared to 10). Similarly, Ruleset E has fewer drawish transitions (74 compared to 94), more winning transitions (24 compared to 14) and more losing transitions (32 compared to 16).

7.2 Playout Behaviour

Given that Ruleset D has fewer drawish positions than Ruleset E, what are the chances that a non-expert player will stumble into a losing move and produce a non-draw result? Table 2 shows the results of 1,000,000 random playouts for each ruleset A to E. Two playout types were implemented with a move limit of $L = 50$ moves per game, to reflect the approximate state space size:

1. **Random:** Strictly random move choice each turn until the game ends or the move limit L is reached.
2. **Greedy:** Strictly random move choice each turn (except that a winning move is made if one exists) until the game ends or the move limit L is reached.

Table 2. Results Over 1,000,000 Playouts.

		White Wins	Black Wins	Draws	Length
Ruleset A	Random	65.35%	26.30%	8.34%	10.80
	Greedy	100.00%	0.00%	0.00%	1.00
Ruleset B	Random	30.54%	52.71%	16.75%	20.58
	Greedy	0.00%	100.00%	0.00%	4.00
Ruleset C	Random	0.00%	0.00%	100.00%	50.00
	Greedy	0.00%	0.00%	100.00%	50.00
Ruleset D	Random	40.82%	36.79%	22.39%	26.13
	Greedy	**54.38%**	**37.89%**	**7.72%**	17.63
Ruleset E	Random	31.93%	33.32%	34.75%	31.78
	Greedy	**41.56%**	**44.39%**	**14.09%**	22.62

Strictly random playouts do not accurately reflect how games between intelligent human players are actually played and do not provide much much insight. The greedy playouts, however, effectively mimic how a beginner might approach the game, playing experimentally and making winning moves if any present themselves. The results from the greedy playouts provide much more insight into the game.

Grredy playouts indicate a 100% win rate for White with Ruleset A, and 100% win rate for Black with Ruleset B, and a 100% draw rate for Ruleset C, all as expected. The results for rulesets D and E suggest much more interesting games. Rulset D has a low draw rate of 7.72% – about half that of Ruleset E – but Ruleset D also suggests a strong first move advantage with a White win rate of 54.38% versus a Black win rate of 37.89%. The win rates for Rulset E are much more balanced at 41.56% and 44.39%.

7.3 Mistake Potential

Chess Grandmaster Savielly Tartakower famously said:

> *The blunders are all there on the board, waiting to be made.*[8]

While mistakes are an embarrassment to the perpetrator that can ruin an otherwise beautiful game from their perspective, they can also inject excitement into the match for the opponent and spectators. Mistakes are in fact crucial to the success of such a simple and drawish game as Mu Torere, as without mistakes every game will end in a draw.

Mistakes can occur in any position that has moves with different game-theoretic values. However, since we want to achieve as many non-draw results as possible, we are most interested in mistakes in which the mover chooses a losing move when a winning or drawish move is available.

Given the set of 46 known positions P, we define a position to have "mistake potential" if it is not a proven losing position but does contain one or more

Table 3. Mistake Potential for Losing Moves by Position.

	P_m/P	**Average Tension**
Ruleset D	13/46	19.57 (±0.09)
Ruleset E	6/46	10.51 (±0.07)

moves whose game-theoretic value is a loss. Table 3 shows the total number of positions with mistake potential P_m for rulesets D and E. Ruleset D has many more positions with mistake potential (13) than Ruleset E (6).

Observing the ratio of losing moves to available moves in each position gives an indication of the *tension* at each position. The second column of Table 3 shows the average tension for both players over all positions, with 95% confidence intervals. Both of these measurements suggest that Ruleset D provides greater potential for non-expert players to make mistakes that produce a non-draw result than Ruleset E.

8 Conclusion

It is incredible that such confusion should exist in the literature over the rules of such a simple game as Mu Torere, especially since the briefest analysis reveals the majority of those rulesets described (A, B and C) to be trivially flawed. The highlights the ease with which errors can be introduced into official accounts of games, and the care with which *any* description of a game must be approached, even those from noted authorities.

Ruleset D – stipulating that the mover cannot win on the first two rounds – is the most prevalent ruleset found in the literature and the most promising ruleset according to the simple analysis performed above. It is conceptually simple, maximises the number of potential wining moves, and gives non-expert players ample opportunity to make mistakes (but it does show indications of strong first move advantage).

Ruleset E – stipulating that the piece being moved must be adjacent to an opponent's piece if it is being moved to the centre point – is a plausible alternative. However, it is conceptually less simple, imposes greater restriction on movement (allowing fewer potential winning moves) and provides less opportunity for non-expert players to make mistakes than Ruleset D.

Future work will involve a full strategic decomposition of Mu Torere and comparison of strategies induced by the two plausible rulesets D and E. It would also be useful to determine if the game is still played, and if so what rules are typically used.

Acknowledgements

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